SIM
SISTEMA INTERAMERICANO DE METROLOGÍA

SIM METROLOGY SCHOOL

Electrical metrology

SPEAKER

Lic. Lucas Di Lillo

Instituto Nacional de Tecnología Industrial (INTI)

Bogotá, Colombia | August 2024

Organized by INM Instituto Nacional de Metrología de Colombia

The International system of units (SI)

The International System of Units (SI)

The recommended practical system of units of measurement is the International System of Units (*Système International d'Unités*), with the international abbreviation **SI**.

From 20 May 2019 all SI units are defined in terms of constants that describe the natural world. This assures the future stability of the SI and opens the opportunity for the use of new technologies, including quantum technologies, to implement the definitions.

The SI is the system of units in which

- the unperturbed ground state hyperfine transition frequency of the caesium-133 atom $\Delta\nu_{Cs}$ is 9 192 631 770 Hz
- the speed of light in vacuum c is 299 792 458 m/s
- the Planck constant h is 6.626 070 15 x 10^{-34} J s
- the elementary charge e is 1.602 176 634 x 10^{-19} C
- the Boltzmann constant k is 1.380 649 x 10^{-23} J/K
- the Avogadro constant N_A is 6.022 140 76 x 10^{23} mol^{-1}
- the luminous efficacy of monochromatic radiation of frequency 540 x 10^{12} Hz, K_{cd}, is 683 lm/W

www.inti.gob.ar/areas/metrologia-y-calidad/si
www.bipm.org/en/measurement-units

The International system of units (SI)

The ampere, symbol A, is the SI unit of electric current. It is defined by taking the fixed numerical value of the elementary charge *e* to be 1.602 176 634 x 10^{-19} when expressed in the unit C, which is equal to A s, where the second is defined in terms of $\Delta\nu Cs$.

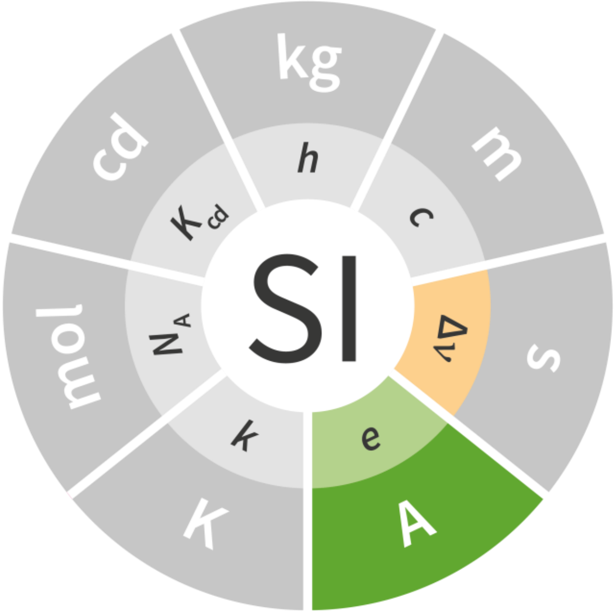
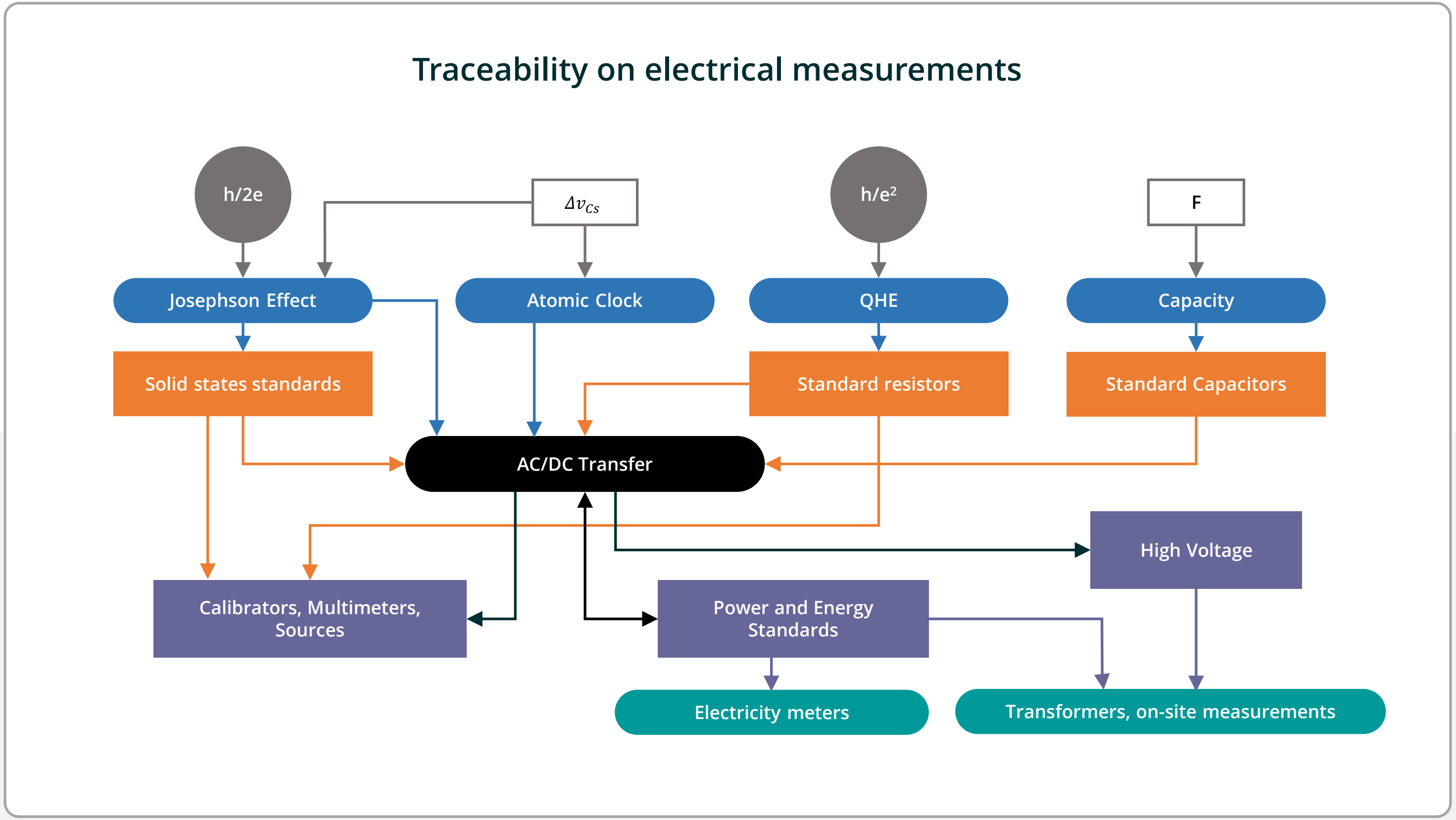


Image from BIPM (www.bipm.org)

Traceability on electrical measurements

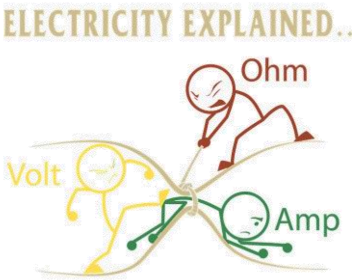


Ampere

In practice, the ampere A can be realized:

a) by using Ohm's law, the unit relation A = V/Ω, and using practical realizations of the SI derived units the volt V and the ohm Ω, based on the Josephson and quantum Hall effects, respectively, as discussed in Secs. 4 and 5 below
b) by using a single electron transport (SET) or similar device, the unit relation A = C/s, the value of e given in the definition of the ampere and a practical realization of the SI base unit the second s
c) by using the relation I = C·dU/dt, the unit relation A = F·V/s, and practical realizations of the SI derived units the volt V and the farad F and of the SI base unit second s

U (Josephson 1V) = 10^{-10}
U (QHE) = 10^{-9}



QHE

V = RI

Josephson Effect

www.bipm.org/utils/en/pdf/si-mep/SI-App2-ampere.pdf
www.bipm.org/utils/common/pdf/CC/CCEM/ccem_guidelines_revisedSI.pdf
www.bipm.org/en/publications/mises-en-pratique/

Ohm

The ohm Ω can be realized as follows:

a) by using the quantum Hall effect in a manner consistent with the CCEM Guidelines [2] and the following value of the von Klitzing constant RK: RK = 25 812.807 459 3045 Ω. This value has been calculated to 15 significant digits. This value follows from the assumption of the accuracy of the equation RK = h/e 2 , Although the quotient h/e 2 can obviously be calculated with any number of digits, this truncated recommended value is in error by less than 1 part in 1015, which is intended to be negligible in the vast majority of applications. In those rare cases where this error may not be negligible, additional digits should be employed. The advantage of recommending a particular value of RK for practical use is that it ensures that virtually all realizations of the ohm based on the quantum Hall effect employ exactly the same value.

b) by comparing an unknown resistance to the impedance of a known capacitance using, for example, a quadrature bridge, where, for example, the capacitance has been determined by means of a calculable capacitor and the value of the electric constant is:

$$\mu_0 = 12.566\,370\,6169(29) \times 10^{-7}\ \mathrm{N\ A^{-2}}$$

www.bipm.org/utils/en/pdf/si-mep/SI-App2-ampere.pdf

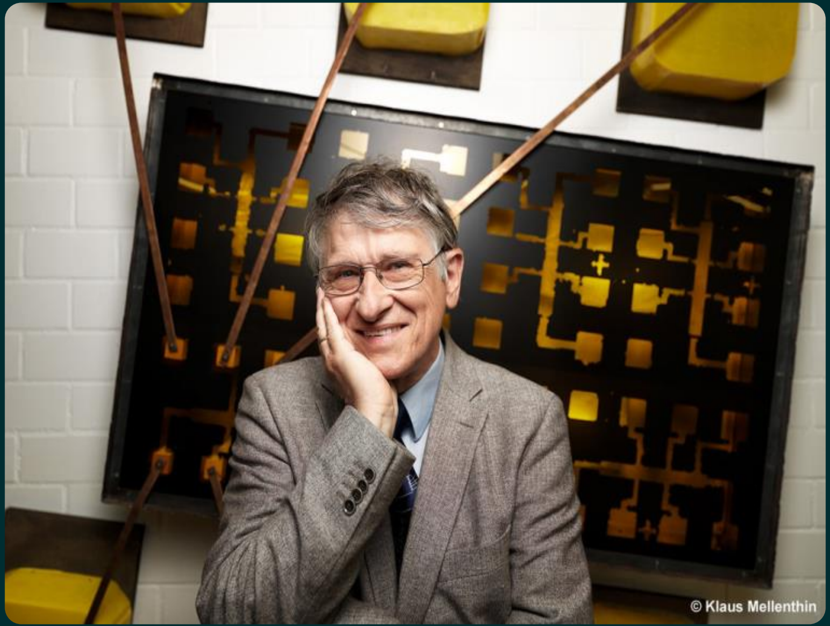
Ohm - Quantum Hall Effect

2DEG + Magnetic field + Low temperatures

$$\rho = \frac{V_{Hall}}{I_{canal}} = \frac{1}{\nu}\frac{h}{e^2} = \frac{1}{\nu}R_K$$

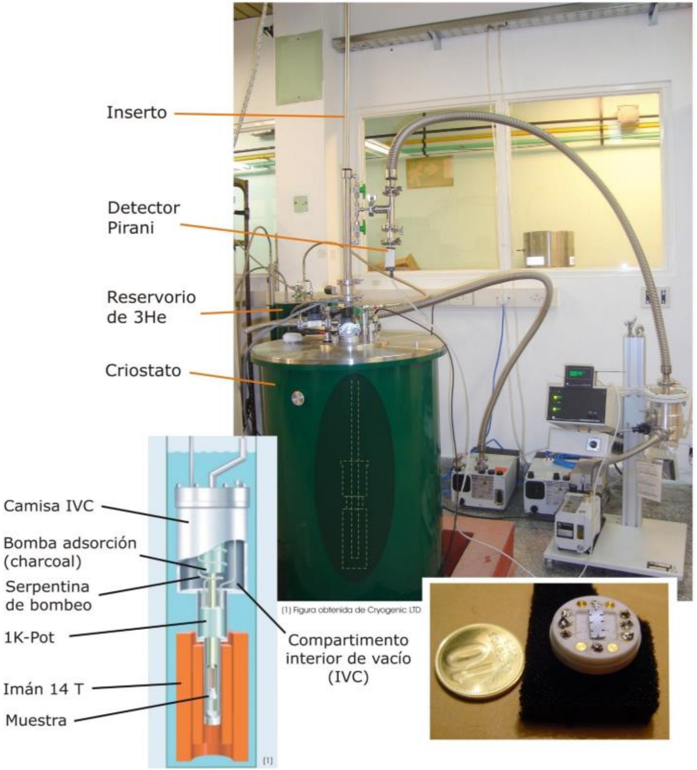
ν number

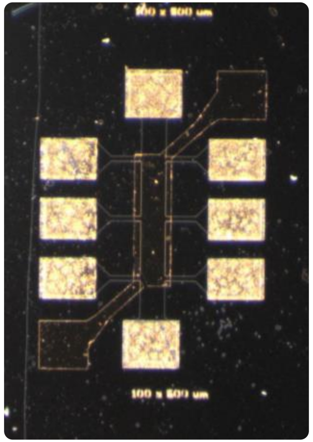
Klitzing, K. V., Gerhard Dorda, and Michael Pepper. "New method for high-accuracy determination of the fine-structure constant based on quantized Hall resistance." *Physical Review Letters* 45.6 (1980): 494.



Discovered by Klaus von Klitzing
(nobel 1985)

Ohm - Quantum Hall Effect



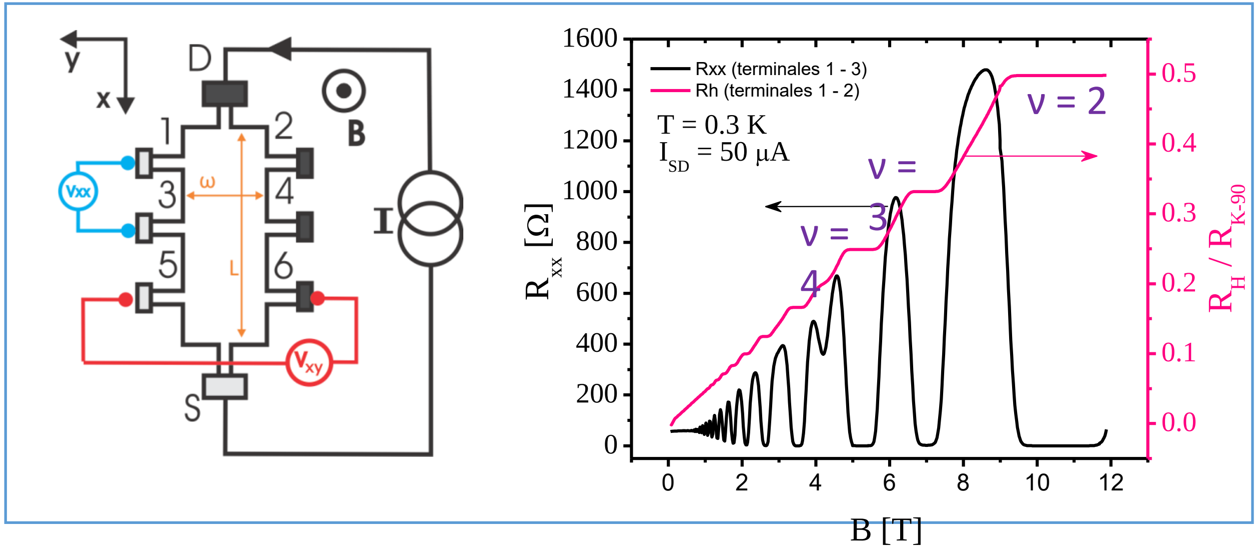


Ohm - Quantum Hall Effect

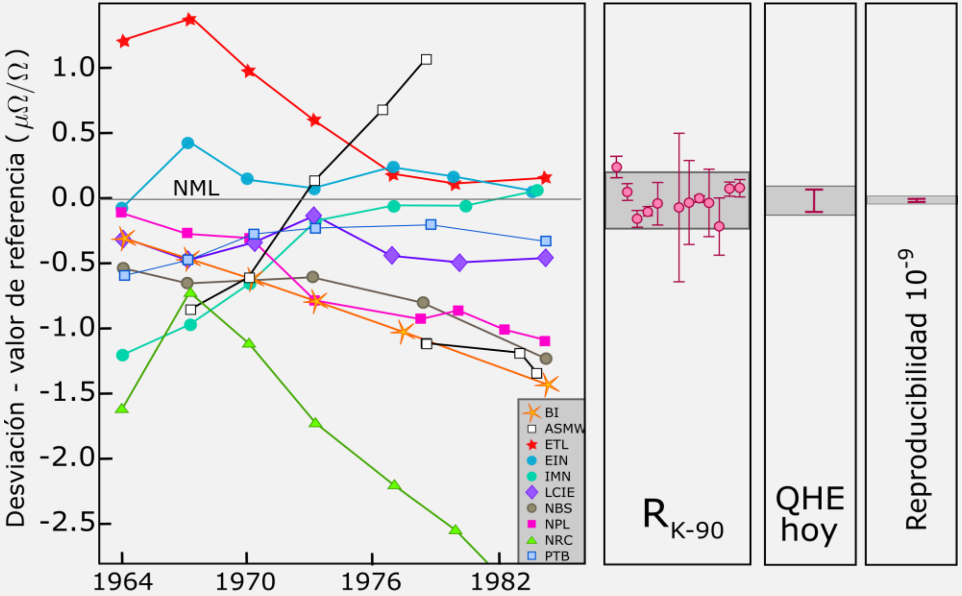
2DEG + Magnetic field + Low temperatures

$$\rho = \frac{V_{Hall}}{I_{canal}} = \frac{1}{\nu}\frac{h}{e^2} = \frac{1}{\nu}R_{K90}$$

ν number



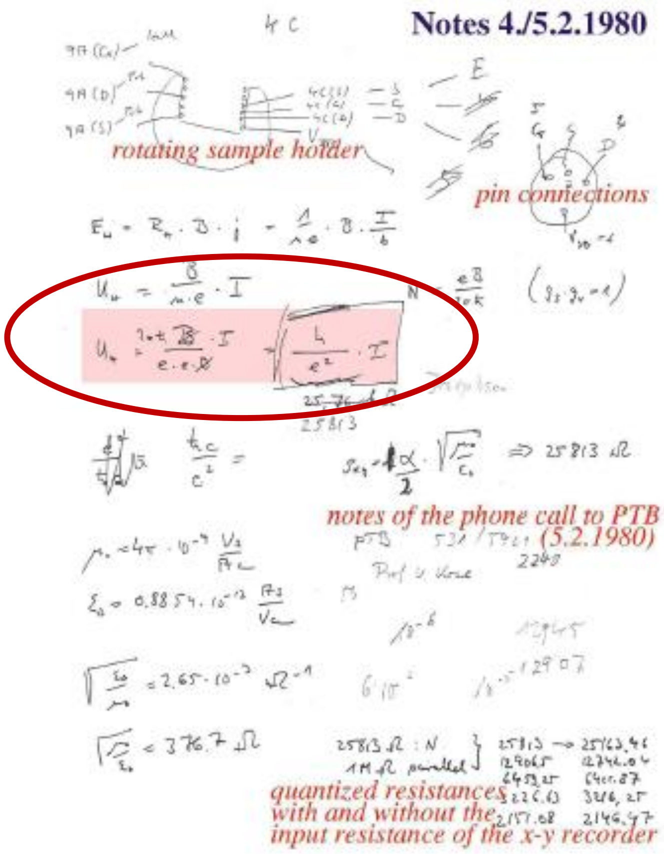
Ohm - Quantum Hall Effect



$$R_K = \frac{h}{e^2} = 25812{,}8074593045\ldots\Omega$$

From J. Ulrich (2017)

Ohm - Quantum Hall Effect



Ohm - Quantum Hall Effect

THE PHYSICAL REVIEW
AND
PHYSICAL REVIEW LETTERS
(PUBLISHED FOR THE AMERICAN PHYSICAL SOCIETY)
BROOKHAVEN NATIONAL LABORATORY, UPTON, LONG ISLAND, NEW YORK 11973

OUR NEW ADDRESS:
1 RESEARCH ROAD
BOX 1000
RIDGE, NEW YORK 11961

June 25, 1980

Dr. K.v. Klitzing
Physikalisches Institut der
Universität Würzburg
D-8700 Würzburg
Federal Republic of Germany

Dear Dr. Klitzing:

The manuscript by

K.v. Klitzing, G. Dorda, and M. Pepper

entitled:

IN REPLY REFER TO: LS1509

Realization of a resistance standard based on fundamental constants

has been reviewed by our referee(s). On the basis of the resulting report(s) we judge that the paper is not suitable for publication in Physical Review Letters in its present form, but might be made so by appropriate revision. Pertinent criticism extracted from the report(s) is enclosed. While we cannot made a definite commitment, the probable course of action if you choose to resubmit is indicated below.

( ) Acceptance, if the editors can judge that all or most of the criticism has been met.
( ) Return to the original referee(s) for judgement.
(✓) Submittal to new referee(s) for judgement.

FIRST PUBLICATION ABOUT THE QUANTUM HALL EFFECT

11 August 1980

Volume 45, Number 6 PHYSICAL REVIEW LETTERS 11 August 1980

New Method for High-Accuracy Determination of the Fine-Structure Constant Based on Quantized Hall Resistance

K. v. Klitzing
Physikalisches Institut der Universität Würzburg, D-8700 Würzburg, Federal Republic of Germany, and Hochfeld-Magnetlabor des Max-Planck-Instituts für Festkörperforschung, F-38042 Grenoble, France

and

G. Dorda
Forschungslaboratorien der Siemens AG, D-8000 München, Federal Republic of Germany

and

M. Pepper
Cavendish Laboratory, Cambridge CB3 0HE, United Kingdom
(Received 30 May 1980)

Recommendations Comité International des Poids et Mesures

(October 4-6, 1988)

recommends

- that 25 812,807 Ω exactly be adopted as a conventional value, denoted by R_{K-90}, for the von Klitzing constant, R_K,

- that this value be used from 1st January 1990, and not before, by all laboratories which base their measurements of resistance on the quantum Hall effect,

- that from this same date all other laboratories adjust the value of their laboratory reference standards to agree with R_{K-90},

- that in the use of the quantum Hall effect to establish a laboratory reference standard of resistance, laboratories follow the most recent edition of the technical guidelines for reliable measurements of the quantized Hall resistance drawn up by the Comité Consultatif d'Electricité and published by the Bureau International des Poids et Mesures,

and is of the opinion

- that no change in this recommended value of the von Klitzing constant will be necessary in the foreseeable future.

Standard resistor

Resistor of 1 Ω L&N tipo Thomas [1]

$\alpha(25\ ^{o}C) \approx 2{,}38\ 10^{-6}\ 1/K$

$\beta(25\ ^{o}C) \approx -5{,}16\ 10^{-7}\ 1/K^{2}$

$\gamma(1023{,}25\ hPa) \approx 6{,}00\ 10^{-9}\ 1/kPa$

BUILDING CHARACTERISTICS

- Wound resistor with 28 turns of bifilar manganin wire
- Cable composition: 83% Cu, 12% Mn, 5% Ni y Fe
- 4 terminals of niquel cooper: 2 for voltaje and 2 for current

Resistor L&N Thomas de 1 Ω

Standard resistor

Resistor of 1 Ω L&N tipo Thomas [1]

$\alpha(25\ ^{o}C) \approx 2{,}38\ 10^{-6}$ 1/K

$\beta(25\ ^{o}C) \approx -5{,}16\ 10^{-7}$ 1/K^2

$\gamma(1023{,}25\ hPa) \approx 6{,}00\ 10^{-9}$ 1/kPa

If we take a simple model that only includes thermal variations

$$R_{(T)} = R(T_0)(1 + \alpha\Delta T + \beta\Delta T^2)$$

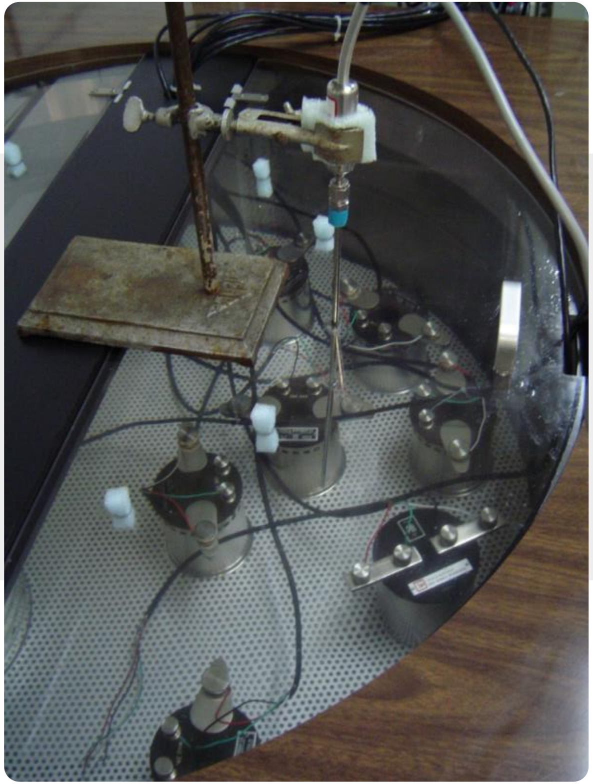
[1] James L. Thomas, "Precision Resistores and Their Measurement", NBS, circ. 470, 1948.

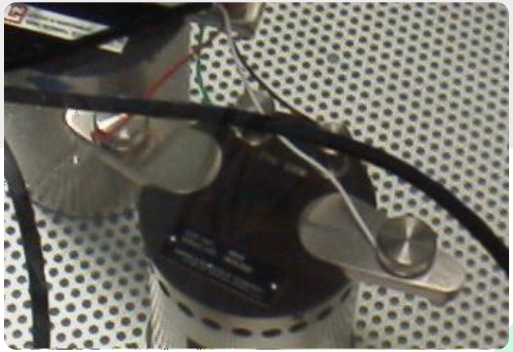
[2] Elmquist, R.E.; Dziuba, Ronald F., "Loading effects in resistance scaling," Instrumentation and Measurement, IEEE Transactions on , vol.46, no.2, pp.322,324, Apr 1997.

Resistor L&N Thomas de 1 Ω

Standard resistors (Thomas)

Stored in a thermostatic oil bath, maintained at 20 °C +/- 1 mK





Standard resistors

Tinsley AC/DC Resistor of 100 Ω

Fluke 742A-100 resistor (100 Ω)
$\alpha(23\ ^{o}C) \approx\ 4{,}00\ 10^{-8}$ 1/K
$\beta(23\ ^{o}C) \approx -2{,}60\ 10^{-8}$ 1/K^2

Ohm

The ohm Ω can be realized as follows:

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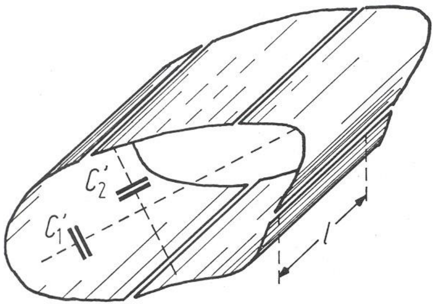
b) by comparing an unknown resistance to the impedance of a known capacitance using, for example, a quadrature bridge, where, for example, the capacitance has been determined by means of a calculable capacitor and the value of the electric constant is:

$$\mu_0 = 12.566\,370\,6169(29) \times 10^{-7}\ \mathrm{N\,A^{-2}}$$

www.bipm.org/utils/en/pdf/si-mep/SI-App2-ampere.pdf

Calculable capacitor



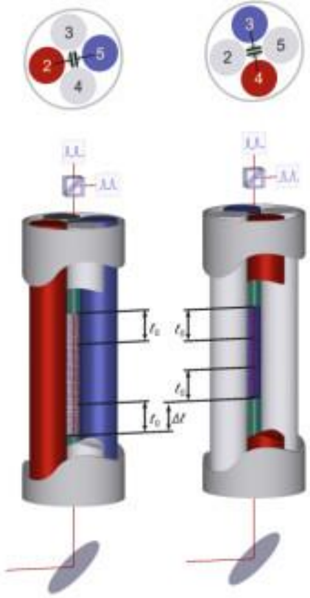
Thompson – Lampard Theorem

$$e^{-\pi C_1'/\epsilon_0} + e^{-\pi C_2'/\epsilon_0} = 1$$

$$C_0' = \frac{\epsilon_0 \ln(2)}{\pi} \approx 2pF/m$$

$$C_1' \approx C_2' \approx C_0'$$

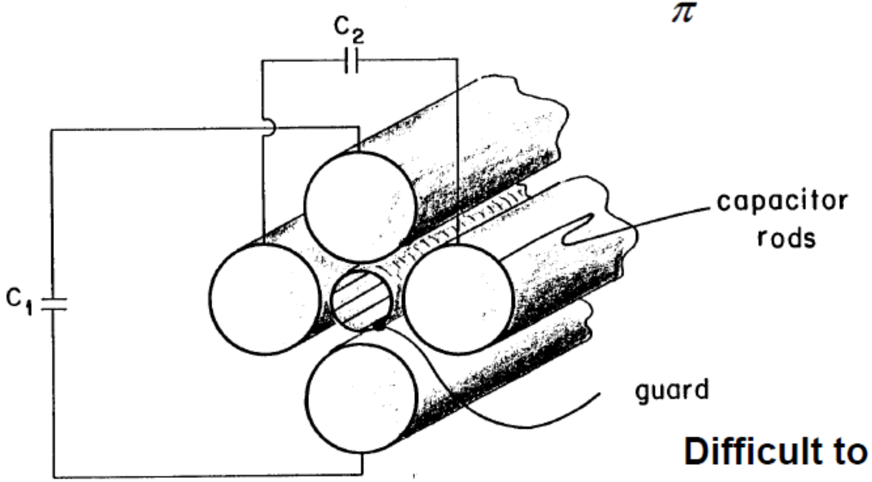
$$\Delta C_0 = \frac{\epsilon_0 \ln(2)}{\pi} \Delta l$$



Calculable capacitor

Thompson Lampard 1956

$$C = \frac{\varepsilon_0}{\pi}\ln(2) \quad F/m$$



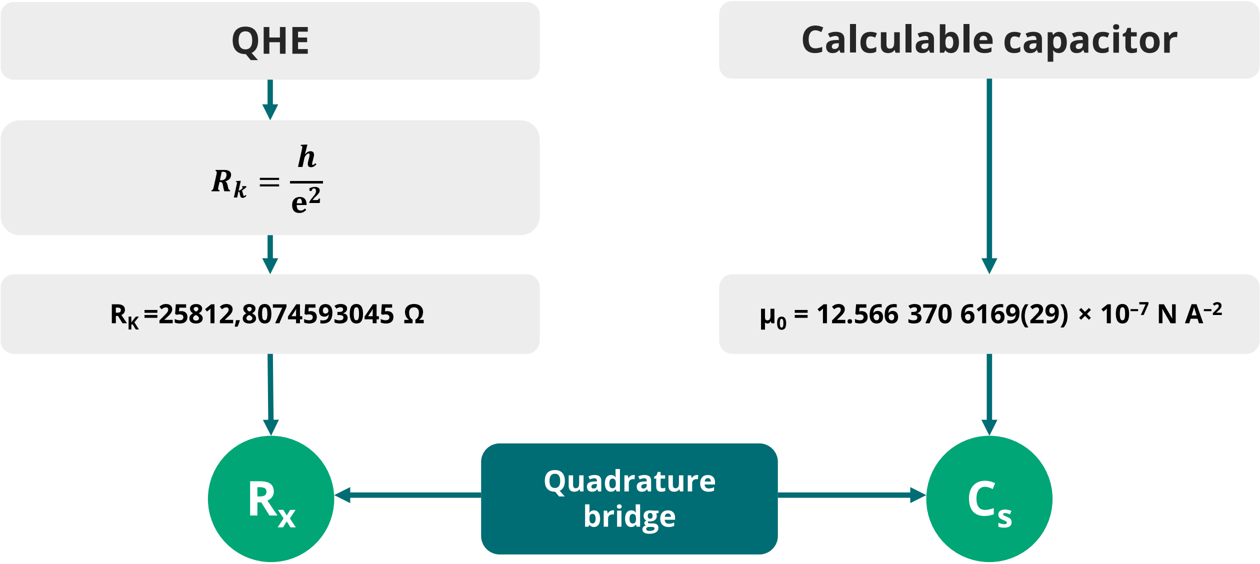
Calculable Capacitor

Difficult to build

10pF @ ~ 0.01 ppm

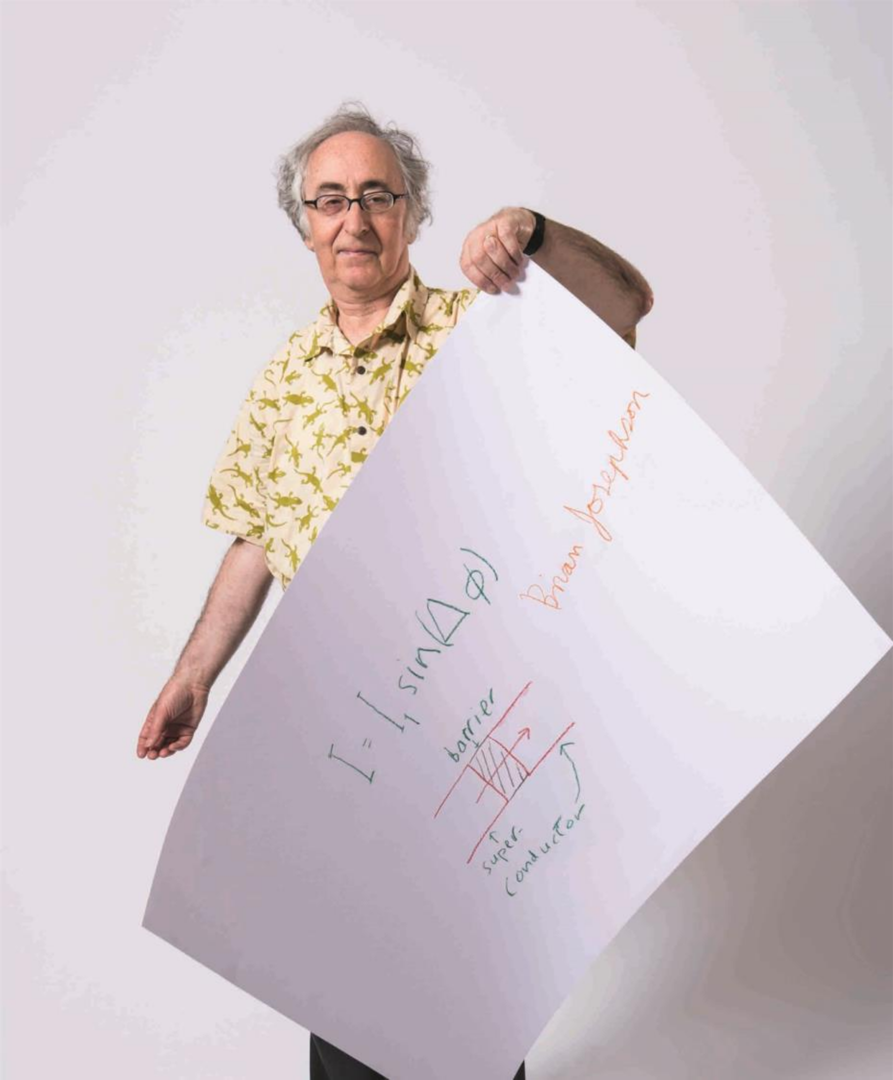
Now used mostly for capacitance

Ohm



www.bipm.org/utils/en/pdf/si-mep/SI-App2-ampere.pdf

Josephson effect (JVS)

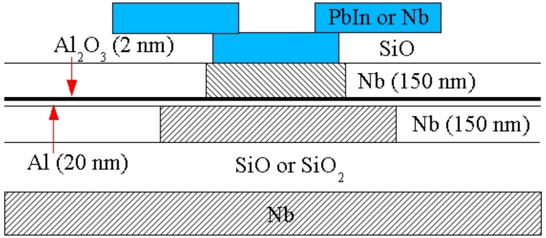


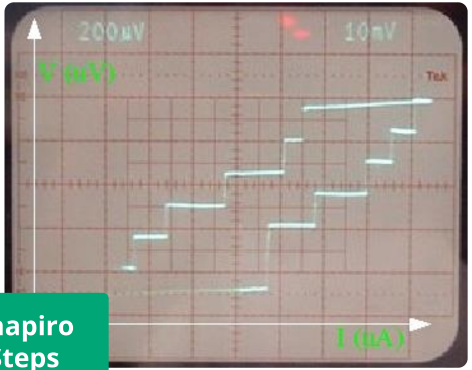
Josephson effect

If microwaves are applied to a joint, then voltage steps appear **“Shapiro steps” (1963)**:

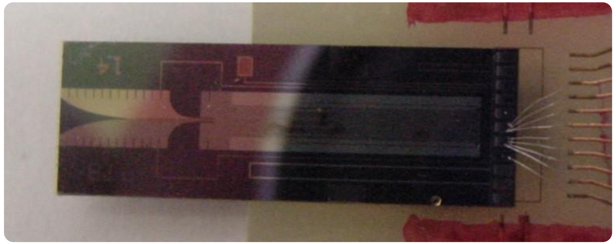
$$V_n = n\frac{h}{2e}f = n\frac{f}{K_J}$$
$$\rightarrow \ (n = \pm 1, \pm 2, \dots)$$

$$K_J = \frac{2e}{h} = 483\,597,848\,416\,984 \text{ GHz/V}$$



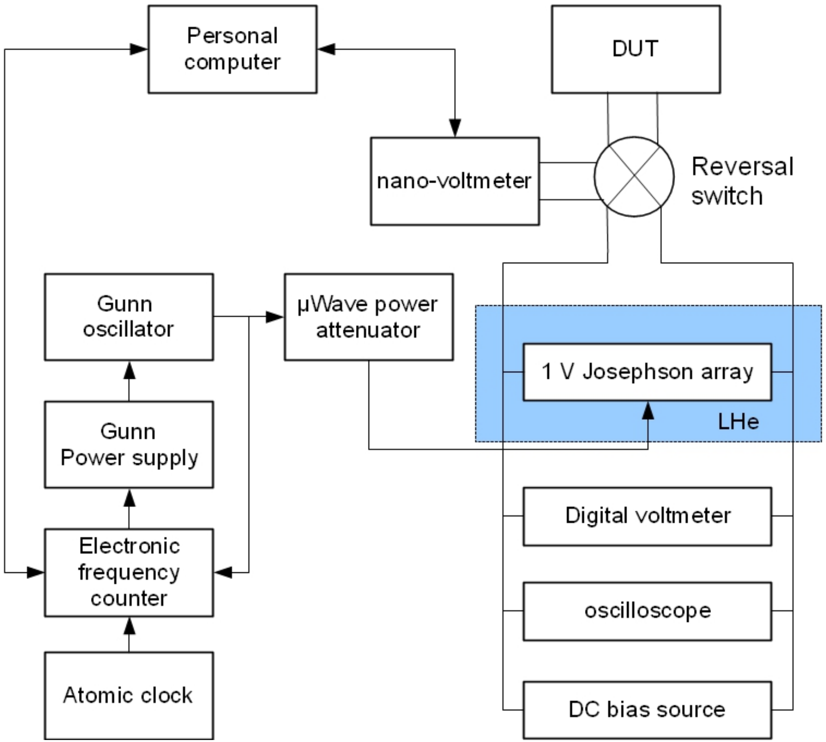


Shapiro Steps



Array Josephson de 1 V (~2000 junturas) Nb/Al_2O_3/Nb (SIS)

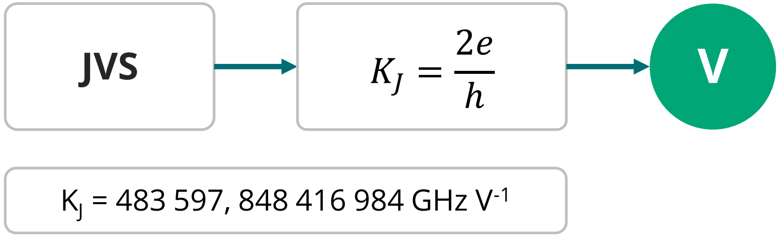
Calibrations using JVS



Volt

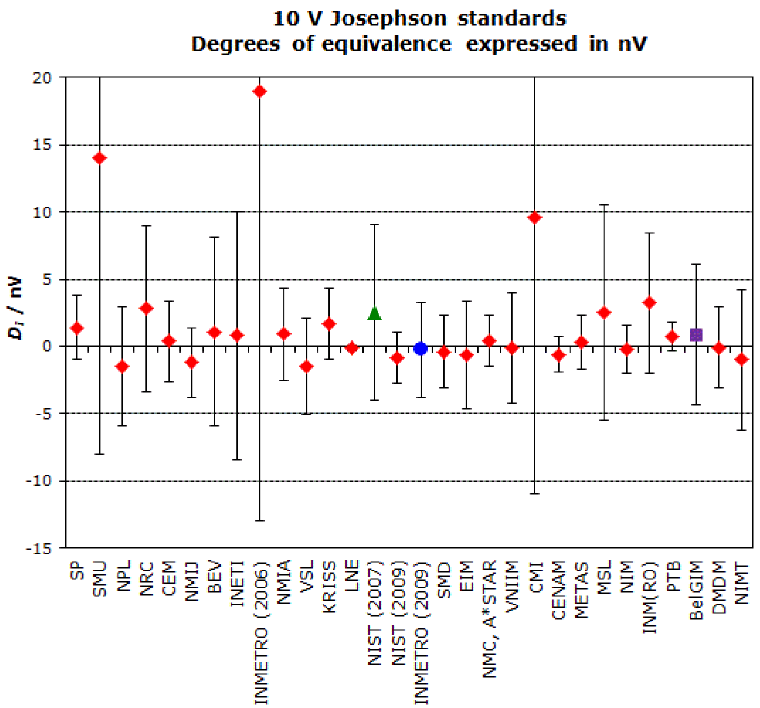
The volt V can be realized using the Josephson effect and the following value of the Josephson constant KJ
KJ = 483 597, 848 416 984 GHz V^{-1}

This value follows from the assumption of the accuracy of the equation KJ = 2e/h, which is strongly supported by a large body of experimental and theoretical works, and the values of *h* and *e.*

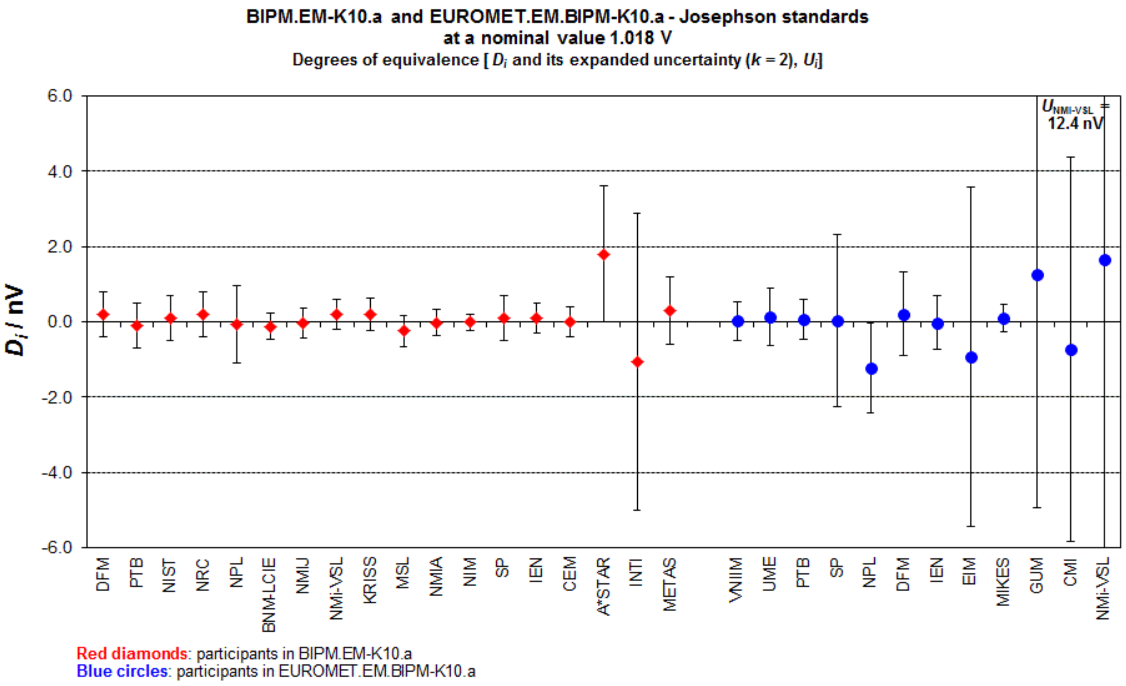


www.bipm.org/utils/en/pdf/si-mep/SI-App2-ampere.pdf

JVS : Reproducibility



JVS : Reproducibilidad



Conventional (CJVS)
Programmable (PJVS)
Josephson Arbitrary Waveform Synthesizer (JAWS)
Quantum Voltage Noise Source (QVNS)

Ampere

In practice, the ampere A can be realized:

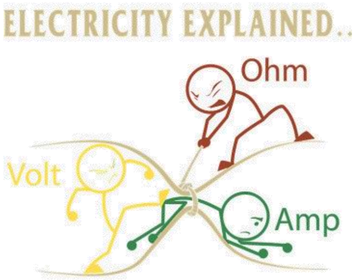
a) by using Ohm's law, the unit relation A = V/Ω, and using practical realizations of the SI derived units the volt V and the ohm Ω, based on the Josephson and quantum Hall effects, respectively, as discussed in Secs. 4 and 5 below

b) by using a single electron transport (SET) or similar device, the unit relation A = C/s, the value of e given in the definition of the ampere and a practical realization of the SI base unit the second s

c) by using the relation I = C·dU/dt, the unit relation A = F·V/s, and practical realizations of the SI derived units the volt V and the farad F and of the SI base unit second s

U (Josephson 1V) = 10^{-10}

U (QHE) = 10^{-9}



QHE

V = RI

Josephson Effect

www.bipm.org/utils/en/pdf/si-mep/SI-App2-ampere.pdf
www.bipm.org/utils/common/pdf/CC/CCEM/ccem_guidelines_revisedSI.pdf
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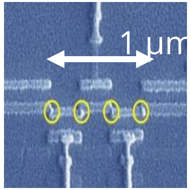
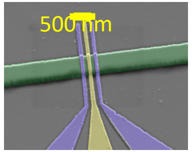
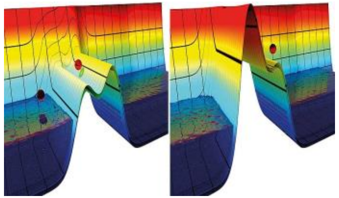
Single Electron Transistor (SET)

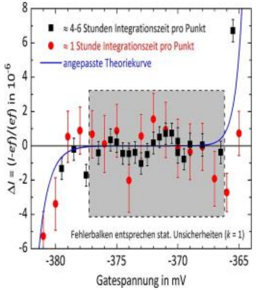


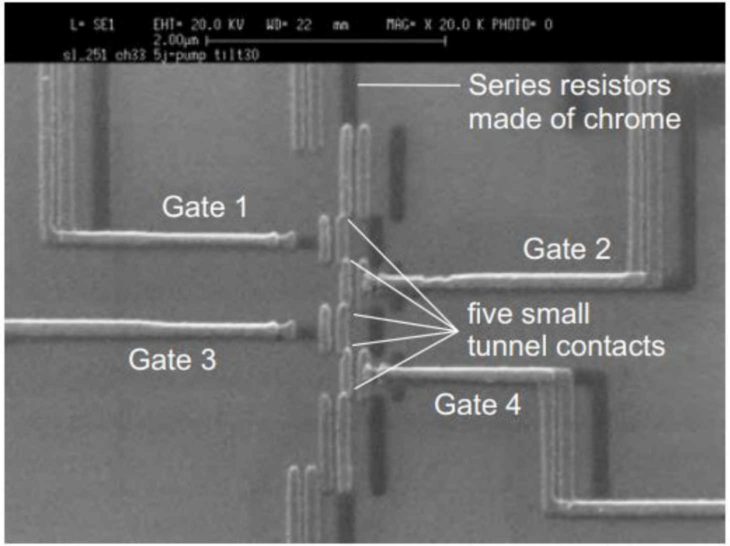




$$I = e \times f \qquad f = 550\ MHz$$

$$\frac{I - e \times f}{e \times f} = (-0.94\ \pm 1.94) \times 10^{-7}$$





https://www.ptb.de/cms/fileadmin/internet/publikationen/ptb_mitteilungen/mitt2012/Heft1/PTB-Mitteilungen_2012_Heft_1_en.pdf#page=60

Single Electron Transistor (SET)

Article | Published: 03 July 2012

Towards a quantum representation of the ampere using single electron pumps

S.P. Giblin, M. Kataoka, J.D. Fletcher, P. See, T.J.B.M. Janssen, J.P. Griffiths, G.A.C. Jones, I. Farrer & D.A. Ritchie

Nature Communications **3**, Article number: 930 (2012) | Cite this article

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doi: 10.1063/1.5048013

PMCID: PMC6604639
NIHMSID: NIHMS1523711
PMID: 31274883

Effect of device design on charge offset drift in Si/SiO_2 single electron devices

Binhui Hu,[1,a)] Erick D. Ochoa,[2] Daniel Sanchez,[2] Justin K. Perron,[2] Neil M. Zimmerman,[3] and M. D. Stewart, Jr.[a),3]

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RESEARCH ARTICLE | MARCH 02 2020

Results and model for single-gate ratchet charge pumping

Roy Murray; Justin K. Perron; M. D. Stewart, Jr.; Antonio L. Levy; Patrick See; Stephen P. Giblin; Jonathan D. Fletcher; Masaya Kataoka; Neil M. Zimmerman

Check for updates

+ Author & Article Information

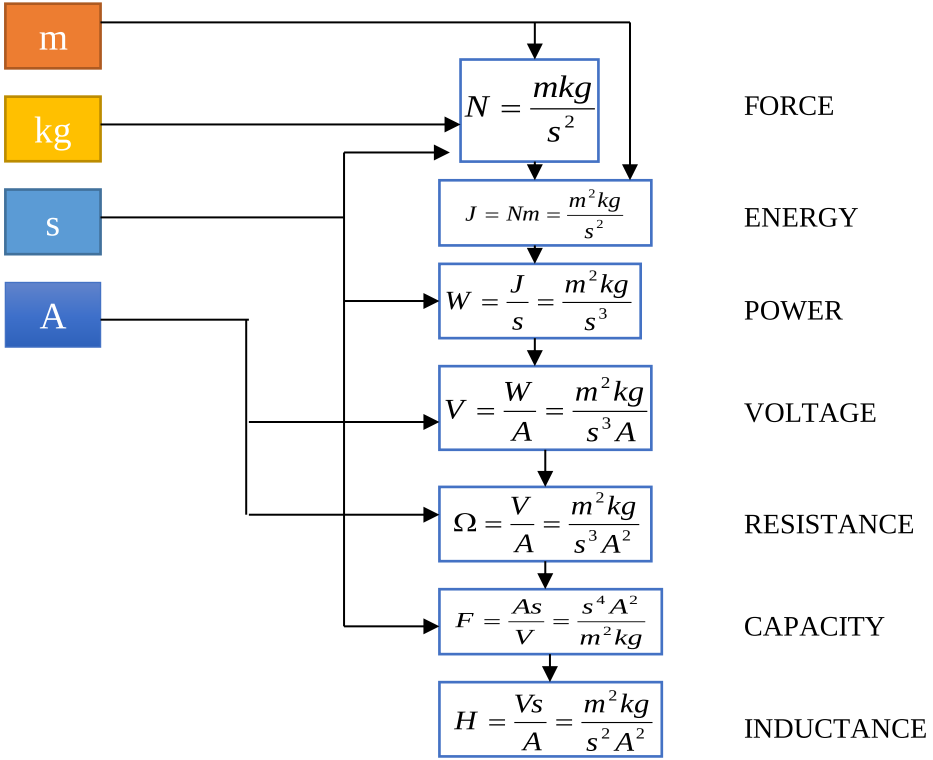
J. Appl. Phys. 127, 094301 (2020)
https://doi.org/10.1063/1.5133967 Article history

Vol. 13, No. 8, pp. 5–10, Aug. 2015. https://doi.org/10.53829/ntr201508fa2

High-speed Single-electron Transfer toward High-accuracy Current Standards

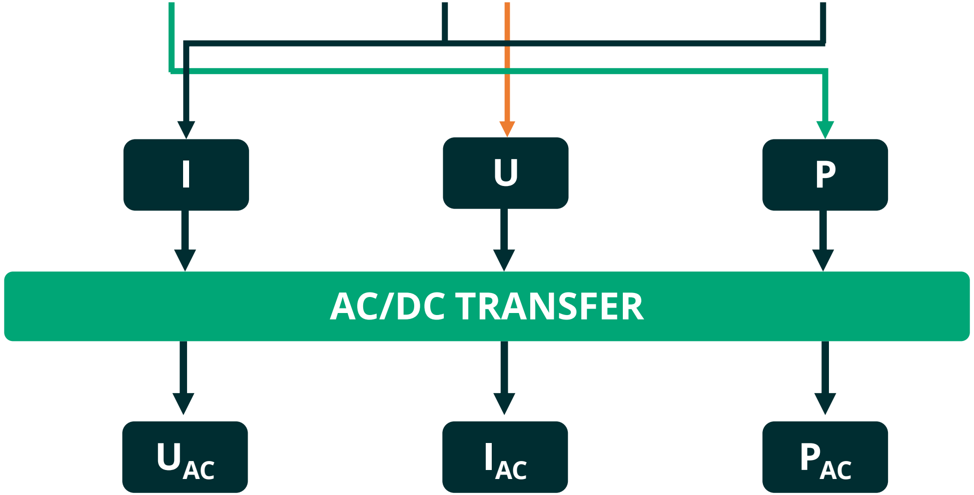
Gento Yamahata, Katsuhiko Nishiguchi, and Akira Fujiwara

Electrical units



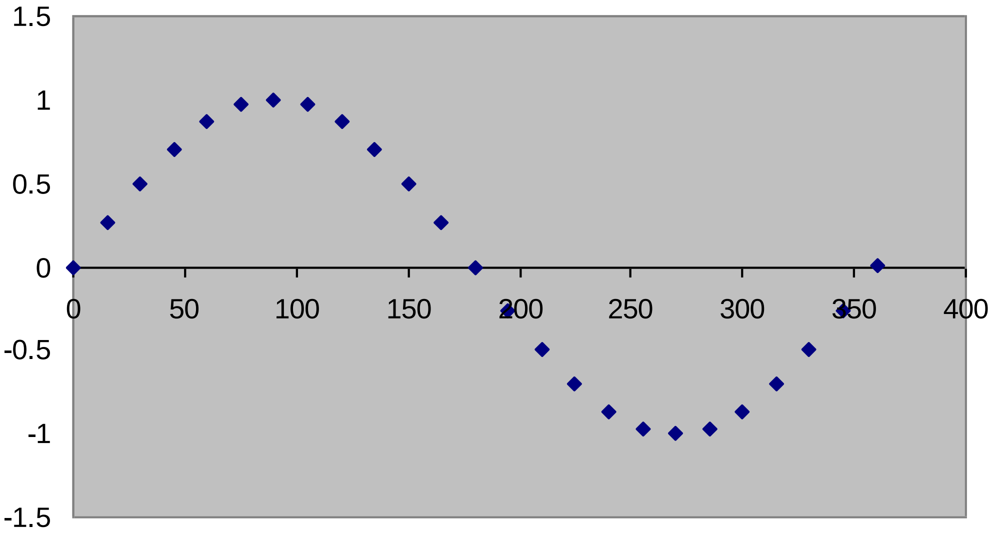

HIGH
AC/DC
VOLTAGE

	Ampere	Volt	Ohm
Definition	A	$V= S^{-3}.m^{2}.kg.A^{-1}$	$\Omega=s^{-3}\ m^{2}\ kg$
Disemination		Zeners	Standard resistors

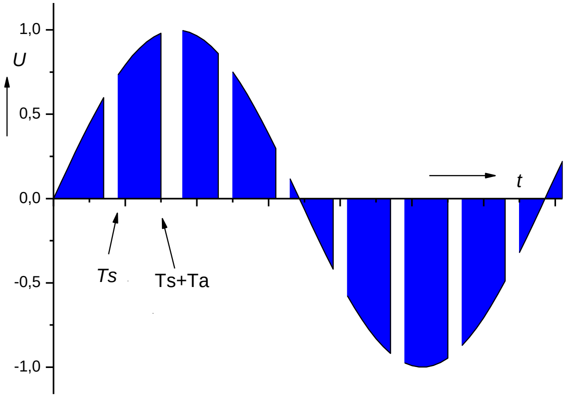


Sampling

$$U_{\text{rms}} = \left(\sum_{i}^{N} u_i^2 \Big/ N \right)^{1/2}$$



Sampling



R.L. Swerlein, “A 10ppm Accurate Digital ac Measurement Algorithm",*H.P internal publication*, Aug 1991

Sampling frequency is not synchronized with the source frequency

$$\Delta = \frac{\sin(wT')}{2wT'}$$

Swerlein´s algorithm choose the best sampling parameters to minimize this error

$$V_k = A_0 + \sum_{j=1}^{Nharm} \left(A_{jk} \sin(2\pi j f t_i) + B_{jk} \cos(2\pi j f t_i) \right)$$

$$y_k = W_k x_k$$

$$x_k = \left(W_k^T W_k \right)^{-1} W_k^T y_k$$

Sampling

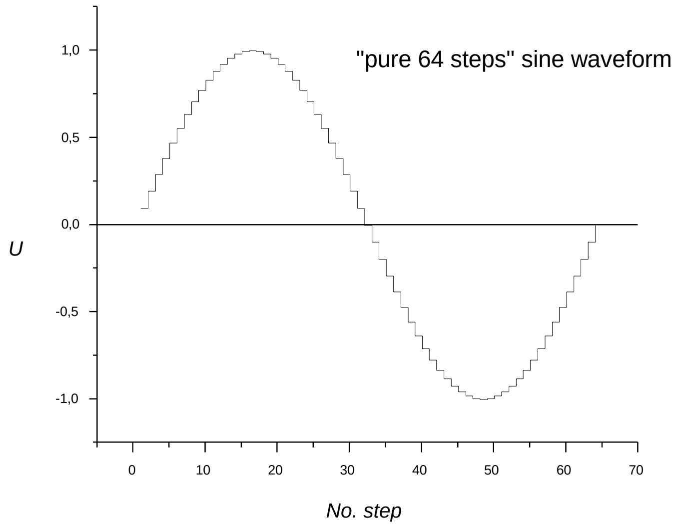

NI PXIe-1071
NATIONAL INSTRUMENTS
PXI

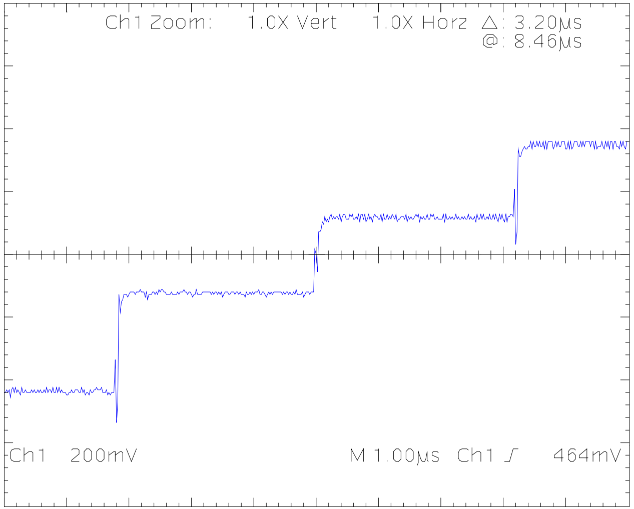

NI PXIe-1073
NATIONAL INSTRUMENTS
PXI


PXI

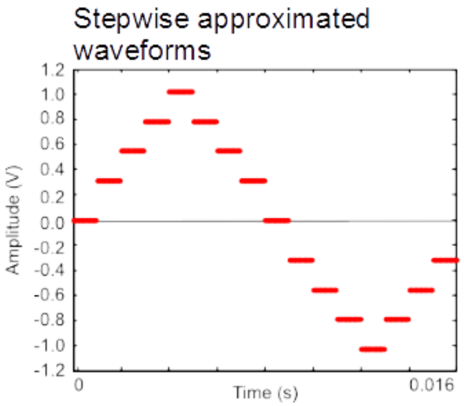

PXI

$$U_{rms} = \left(\sum_{i}^{N} u_i^2 / N \right)^{1/2}$$





Programable josephson (PJVS)

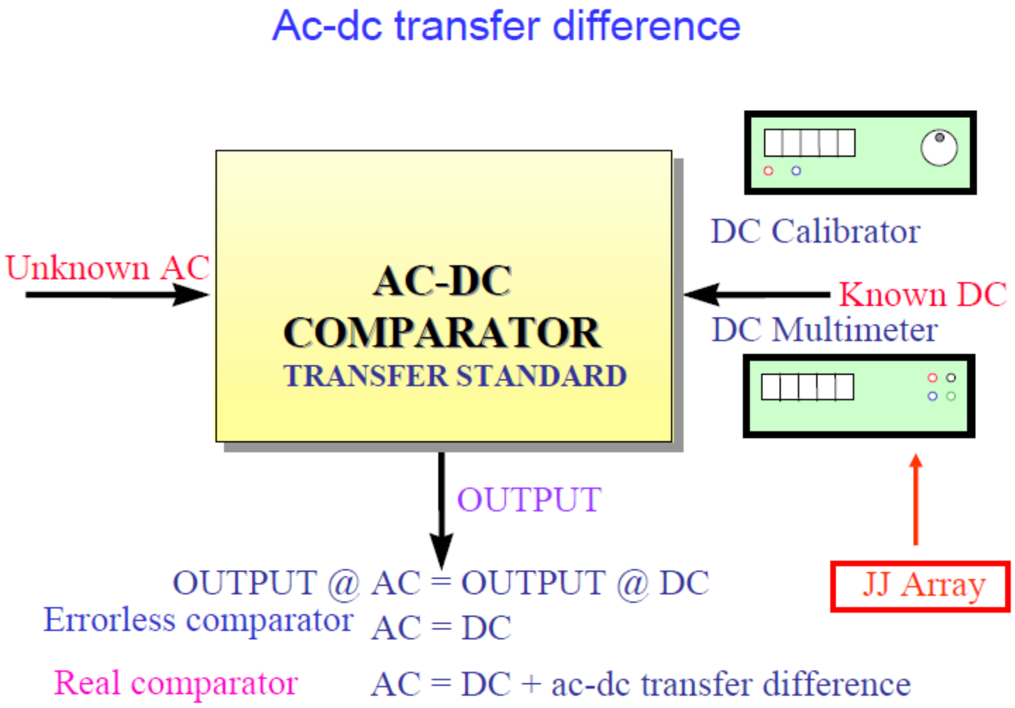


$$V_J = n_{JJ} \frac{h}{2e} f$$

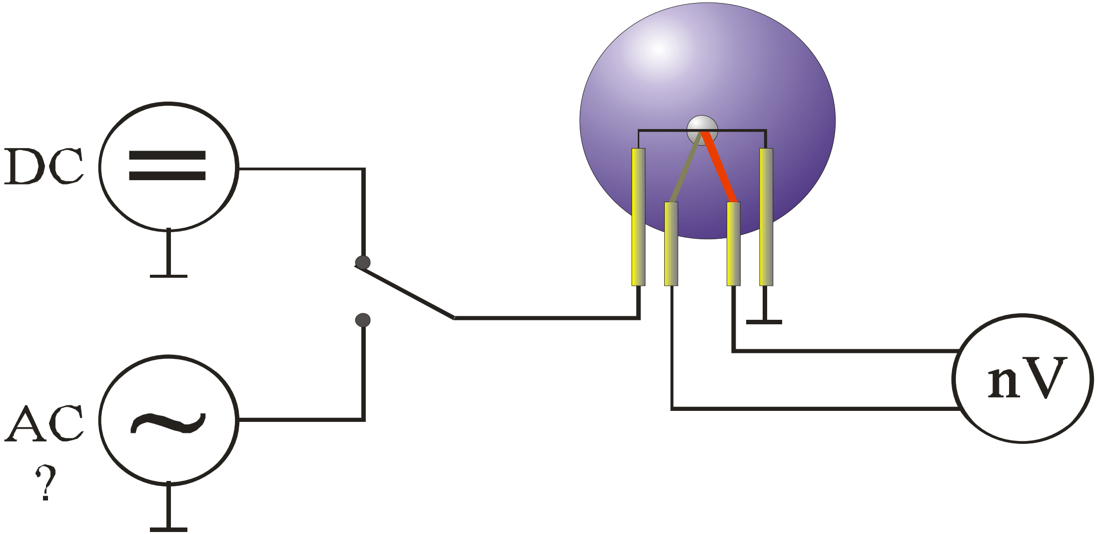
$$u_{V_J} \simeq 10^{-10}$$

No Noise

AC/DC transfer difference

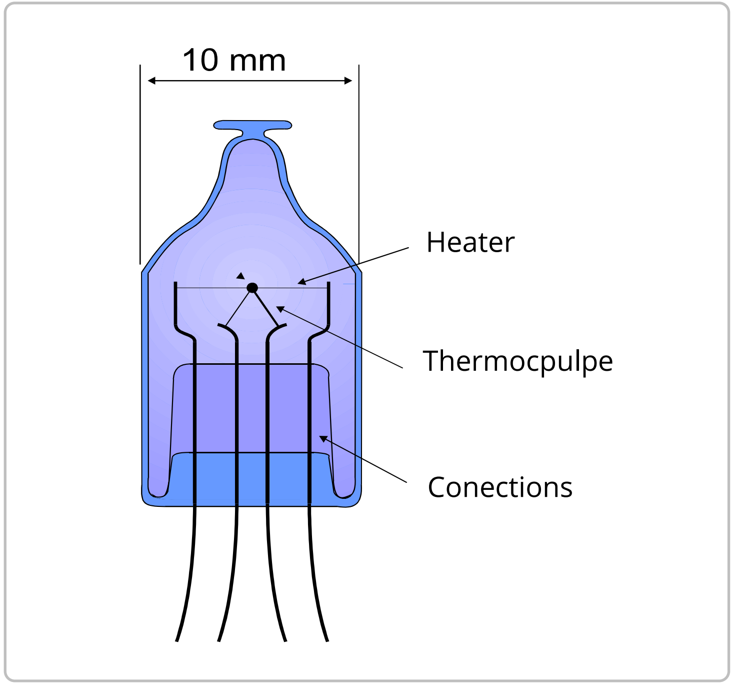


AC/DC transfer difference



AC/DC Transfer Difference $\delta = \frac{U_{ac} - U_{dc}}{U_{dc}}$ When $U_{ac}^{o} = U_{dc}^{o}$

AC/DC transfer difference (Single junction)



Heater → Ni-Cr
TC: Cu- Constantan
ΔT= 150 K

$$Uo = k \cdot Ui^n$$

AC/DC transfer difference (Single junction)

Typical parameters of SJTC

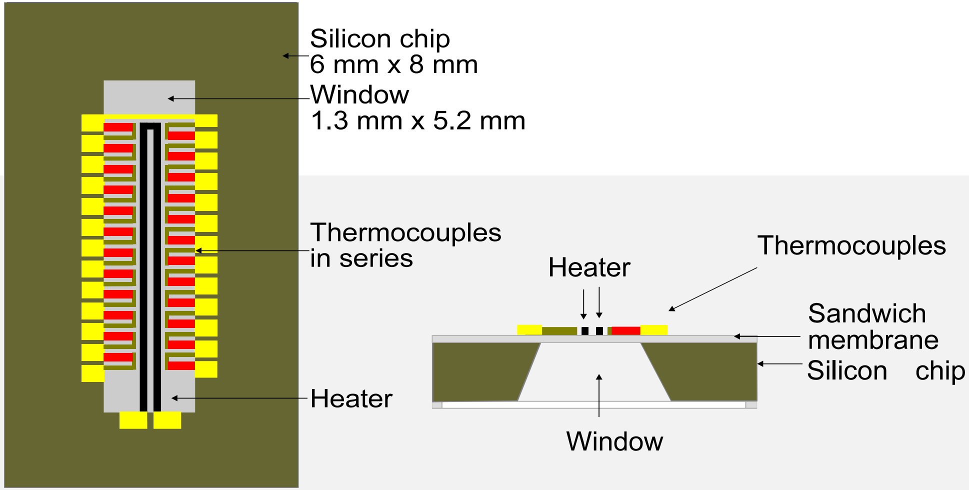
Heater resistance	Rh	90 Ω
Rated input current	I	5 mA
Rated input voltage	V	0.45 V
Output voltage	E	7 mV
Output thermocouple resistance	Rtc	16 Ω
Time constant	τ	1-4 s
Frequency range (UHF)	f	up to 1GHz
Dc reversal error (selections)		0.005%-0.2%

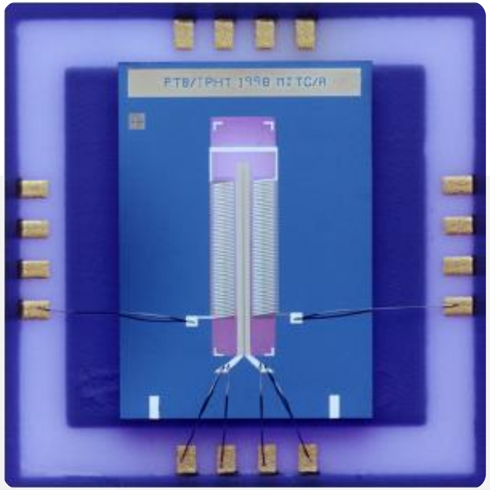
Temperature increment of the heater >150 deg

Very sensitive to overload (<150% - 200% destructive)

Sensitive to voltage stress between input and output circuits (usually tested at 100 V) – always ground the thermocouple output.

AC/DC transfer difference (Planar Multi junction)



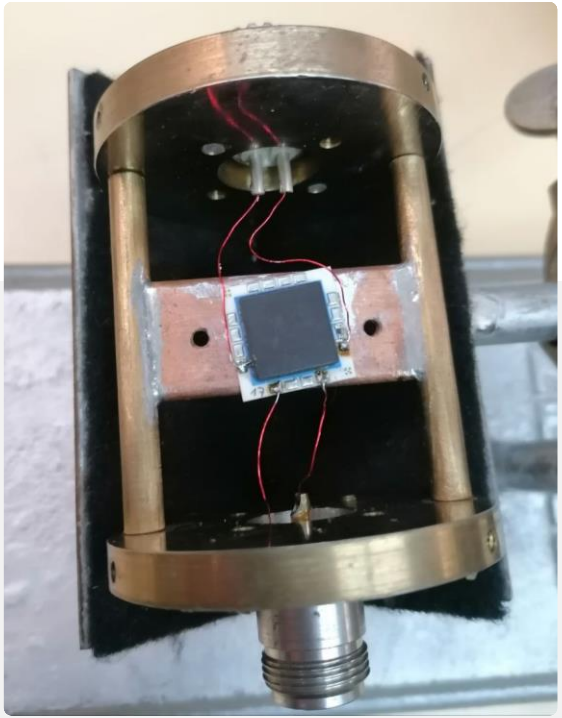


AC/DC transfer difference (Planar Multi junction)

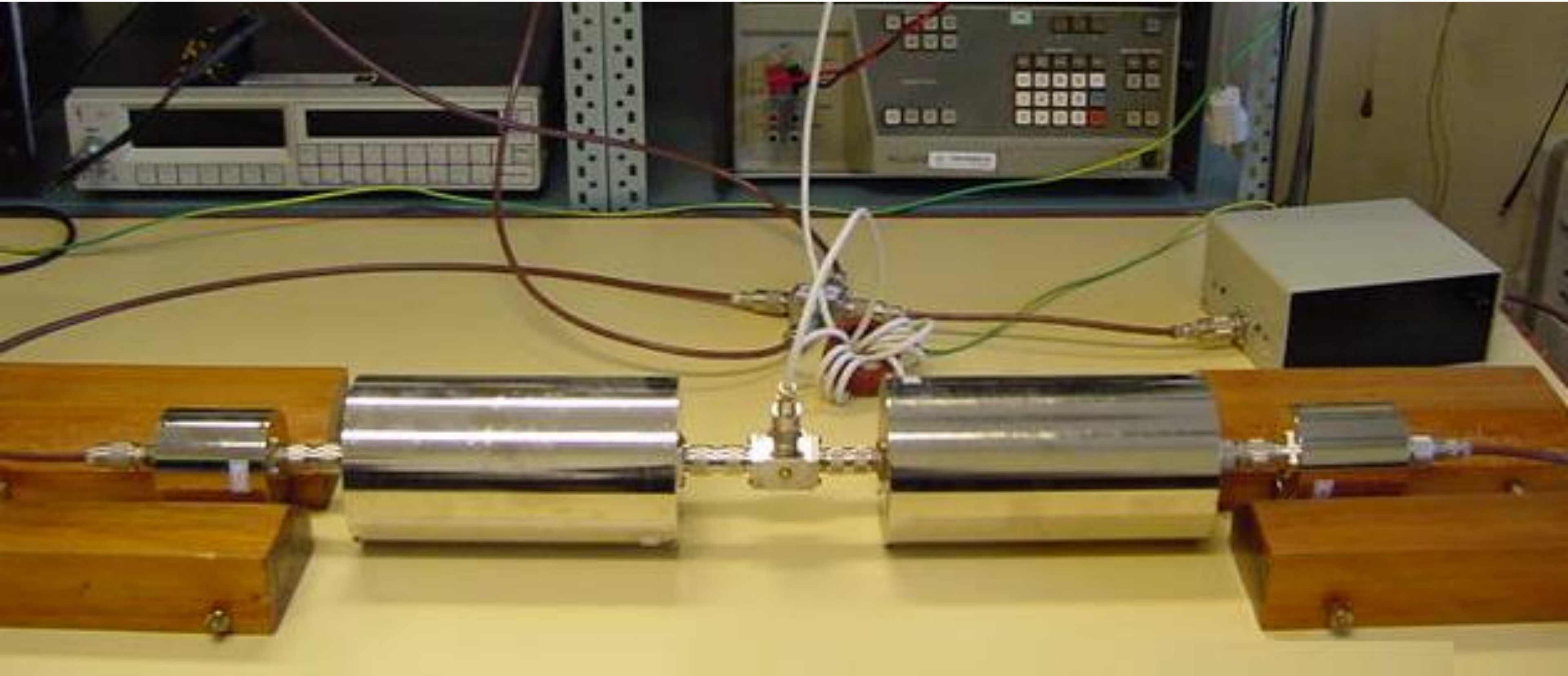
Specifications of PTB Planar MJTC

Chip size 6x8 mm -	carrier size 18x18 mm
Sandwich membrane Si_3N_4-SiO_2-Si_3N_4	0.8 µm
Heater material	NiCrSi
Heater resistance	(90, 180, 400, 900) Ω
TC of the heater	1x10-6 1/K
Number of thermocouples	100
Thermoelectric material	Bi-Sb
Sensitivity in air	15 V/W
Thermocouple resistance	10 kΩ
Rated input voltage	(1; 1,5; 2; 3) V
Output voltage	(100 to 200) mV
Reversal difference	<1 part in 10^6
Time-constant with obelisk	1.8 s

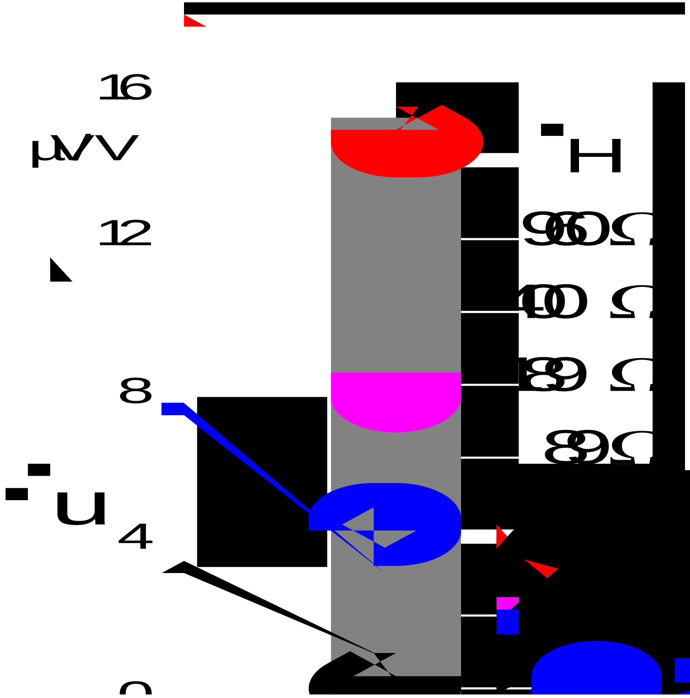
AC/DC transfer difference



AC/DC transfer difference (set up)



AC/DC transfer difference



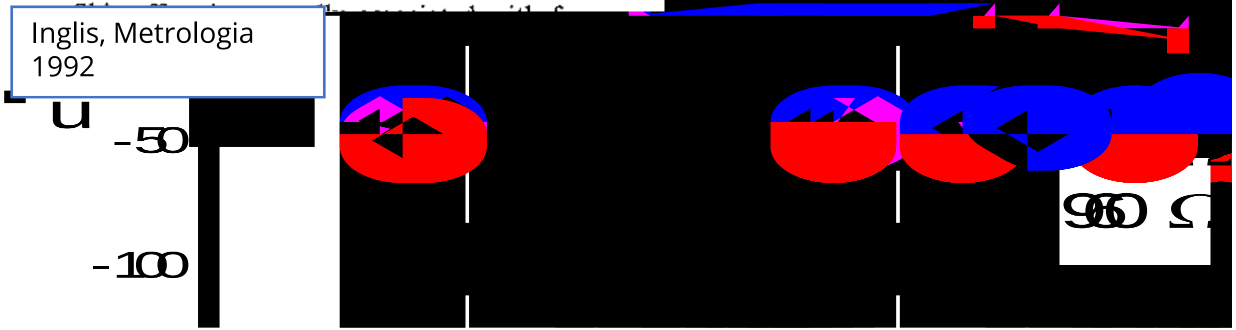
2.2.2 Frequency-dependent transfer errors

Low frequency. At frequencies below approximately 10 Hz, errors arise due to performance limitations of the TCs. They arise because the thermal inertia of the heater is insufficient to average out the twice frequency variation in the joule heat or I^2R input power, and this leads to a twice frequency output signal from the thermocouple [14]. Ideally, with a true square law TC input/output characteristic, this would present no problem if the output emf from the thermocouple could be integrated at the detector and compared with the output on dc [16]. However, the input/output characteristic of a practical converter is not a perfect square law, and herein lies a source of error.

Inglis, Metrologia 1992

AC/DC transfer difference

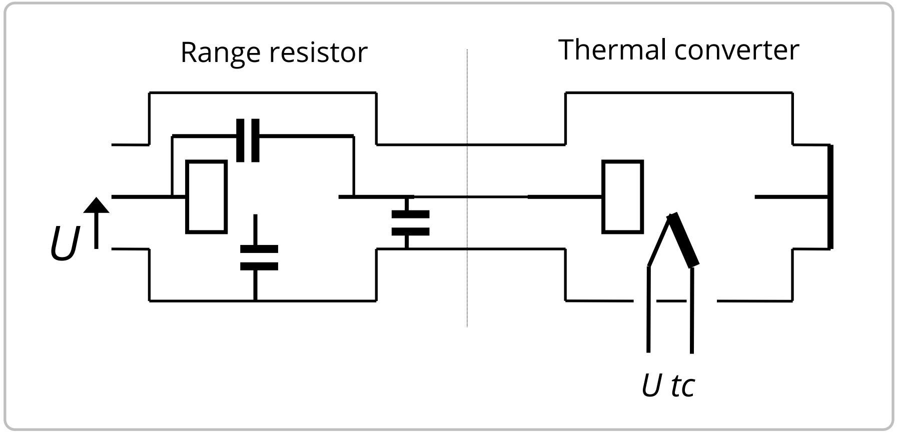
High frequency. At higher frequencies, capacitance, dielectric loss, inductance, skin-effect and standing waves all become significant, and in the presence of these effects the joule heat dissipated in the heater for a given rms ac input is different from that for an equivalent dc value. Inductance and skin-effect associated with circuit wiring, the TC leads and the TC heater tend to limit performance on low-voltage ranges, while capacitance, dielectric loss and voltage coefficients associated with the range resistors determine performance characteristics at higher voltages.



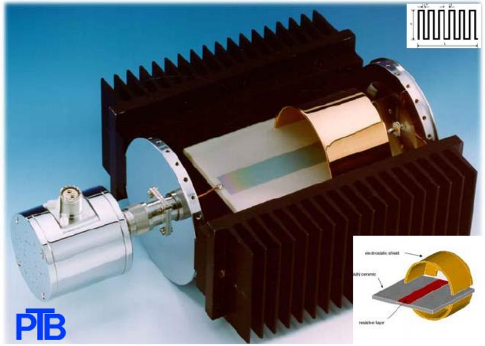
Inglis, Metrologia 1992

AC/DC transfer difference (Voltage)

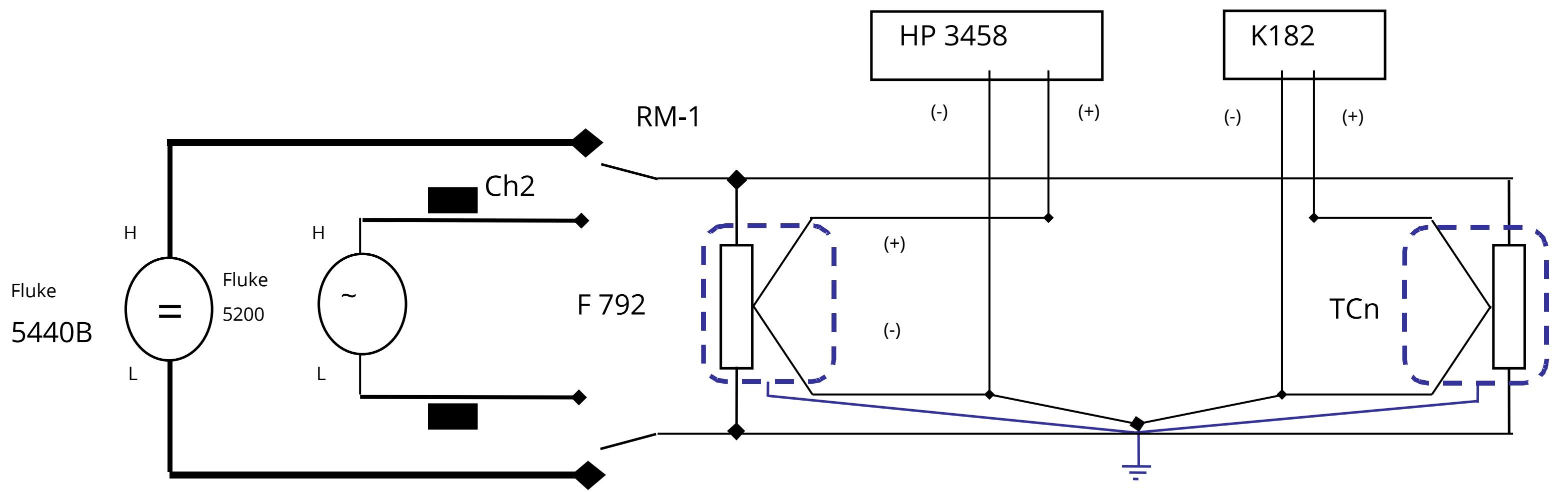
Capacitance that affects the AC-DC transfer difference



PTB HV resistor

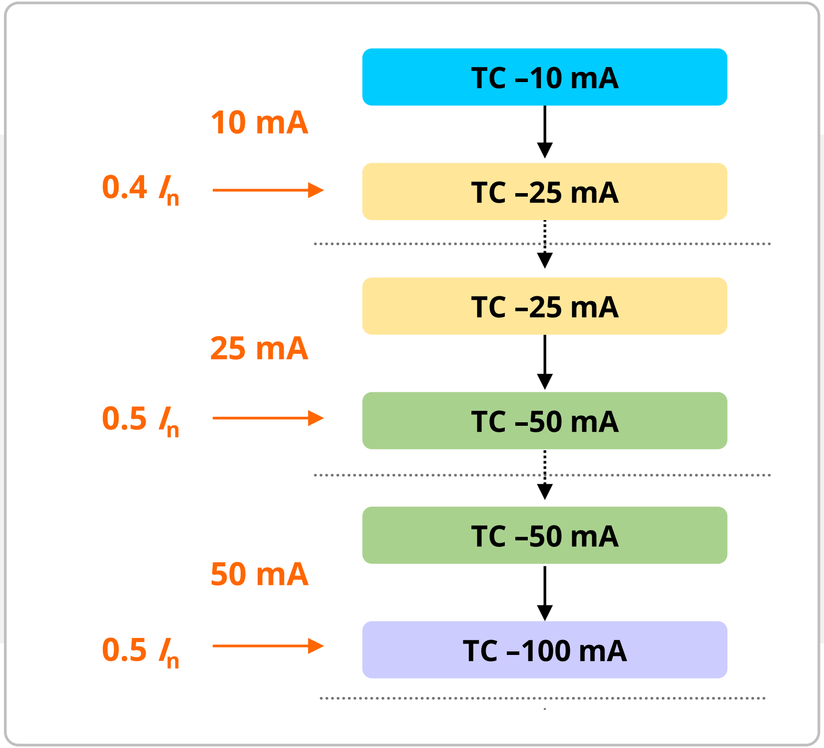


AC/DC transfer difference (Measuring set up)

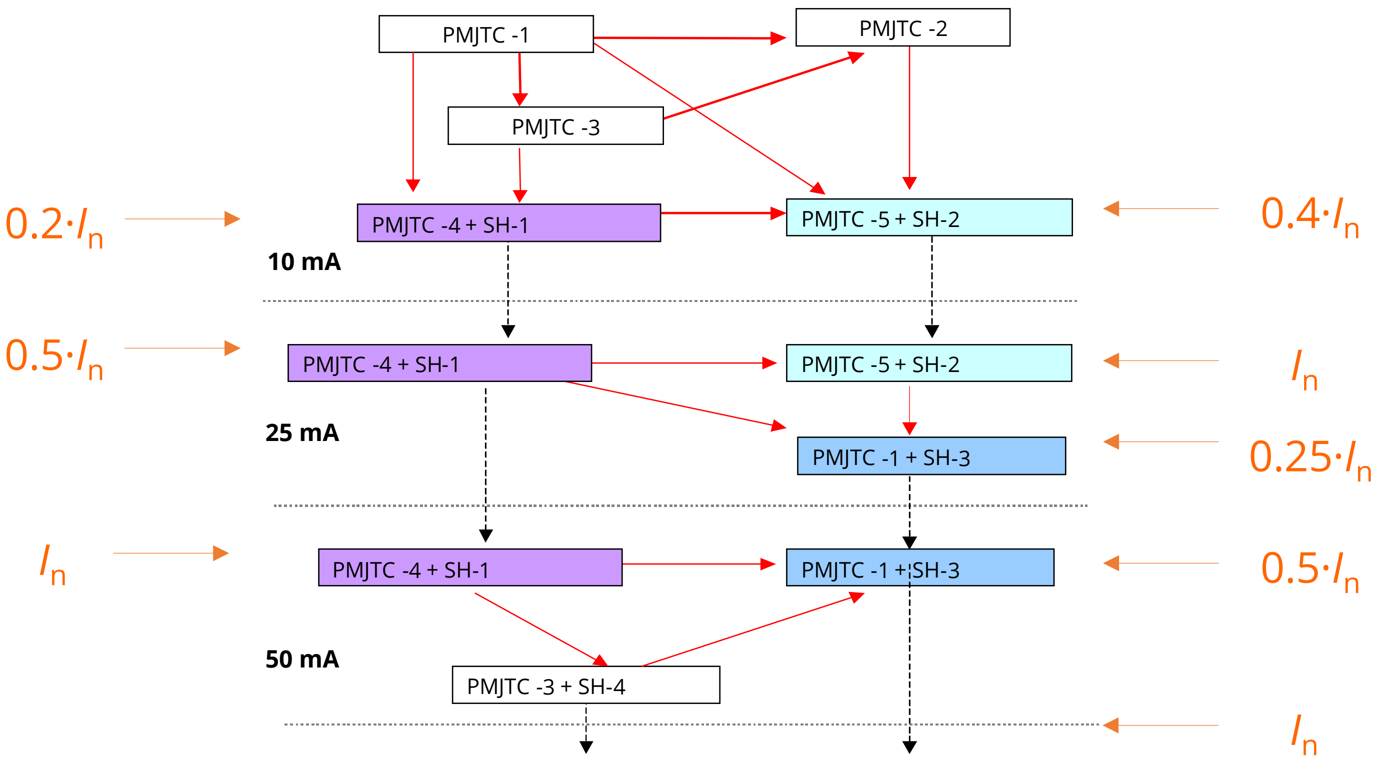


AC/DC transfer difference (step up)

Step-up process



AC/DC transfer difference (step up)



AC/DC transfer difference (calculations)

AC-DC Difference

$$\delta = \frac{U_{ac} - U_{dc}}{U_{dc}} \quad \text{when} \quad U^{o}_{ac} = U^{o}_{dc}$$

AC, DC+, AC, DC-, AC

$$\frac{\overline{U}_{0acx}}{\overline{U}_{0acs}} \qquad \frac{\overline{U}_{0dcx}}{\overline{U}_{0dcs}}$$

$$\text{Convergencia} = \frac{\left(\overline{U}_{0acs} - \overline{U}_{0dcs}\right)}{\overline{U}_{0dcs}}$$

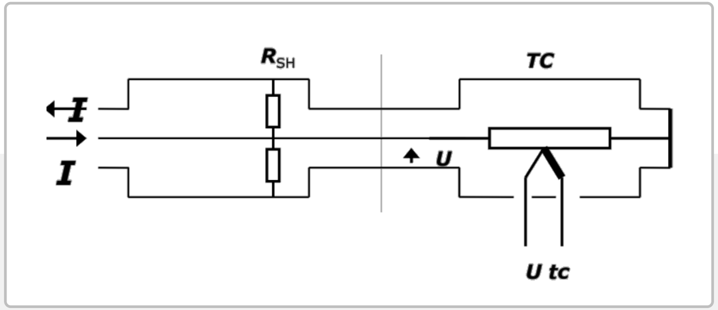
> 50 ppm ➔change the Dc current (voltaje)

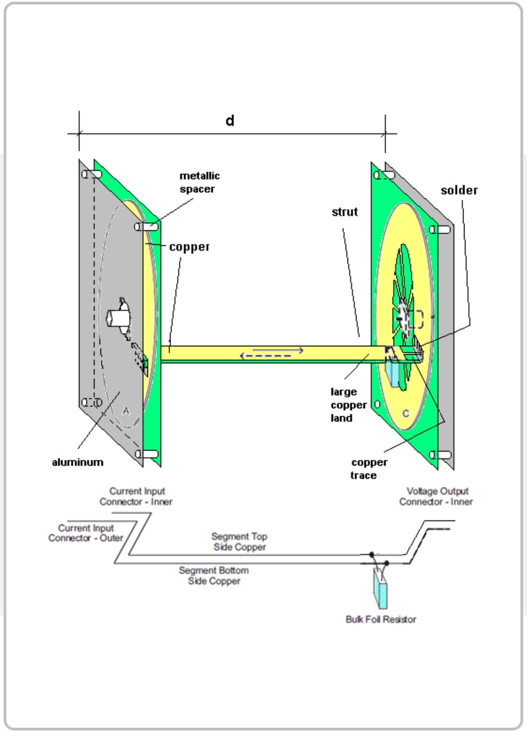
< 50 ppm ➔ the value

$$\delta_{\mathrm{d}} = \frac{\left(U_{\mathrm{odcx}} - U_{\mathrm{oacx}}\right)}{n_x . U_{\mathrm{odcx}}} - \frac{\left(U_{\mathrm{odcs}} - U_{\mathrm{oacs}}\right)}{n_s . U_{\mathrm{odcs}}}$$

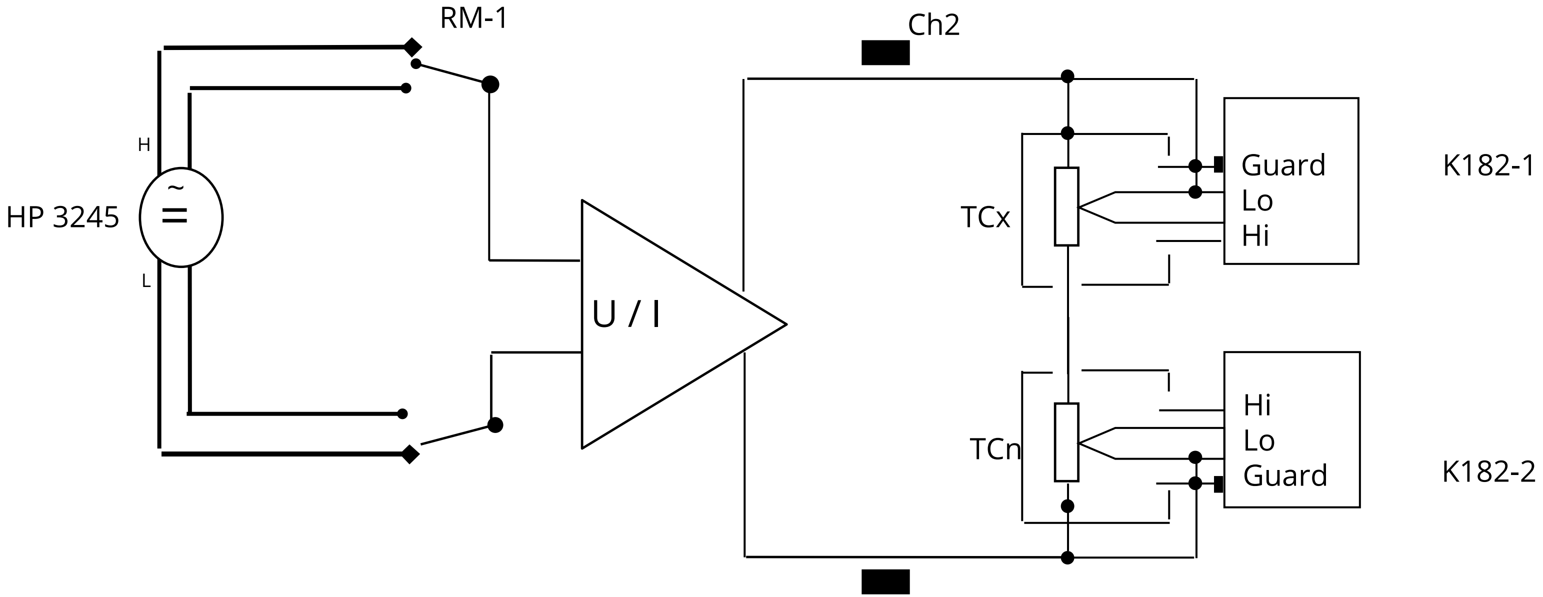
$$\delta_x = \delta_d + \delta_s$$

AC/DC transfer difference (Shunts)

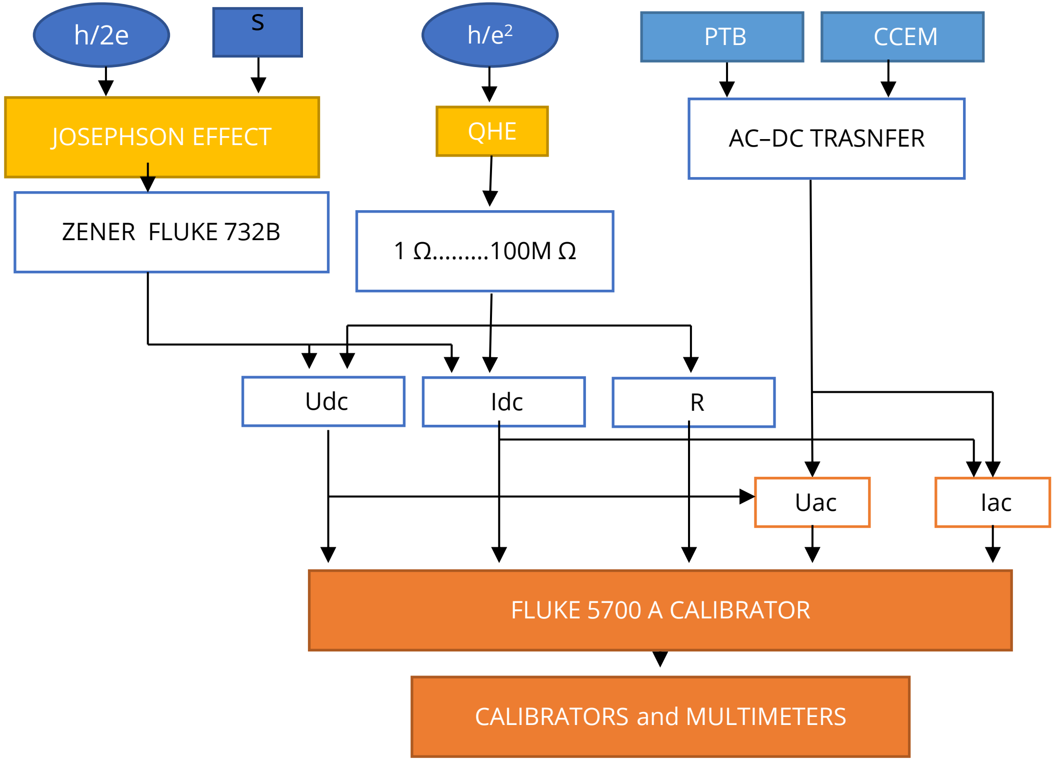




AC/DC transfer difference (set-up)

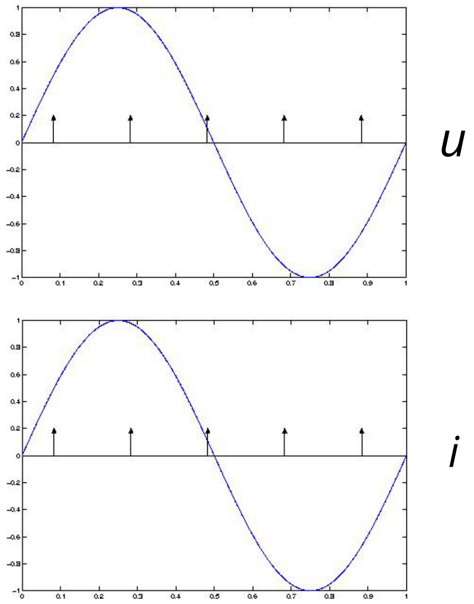


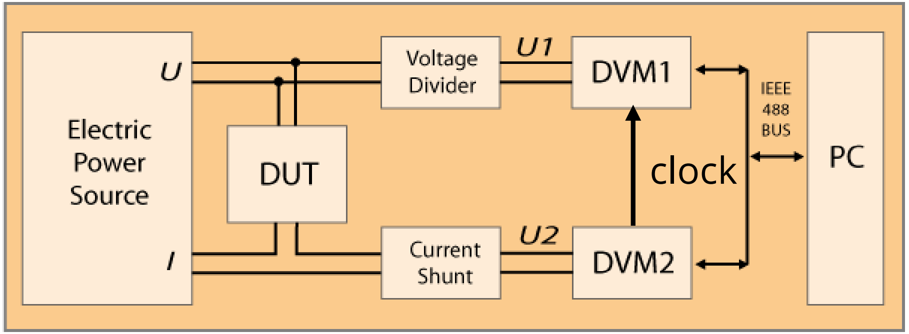
Traceability



Equipmenets

Power

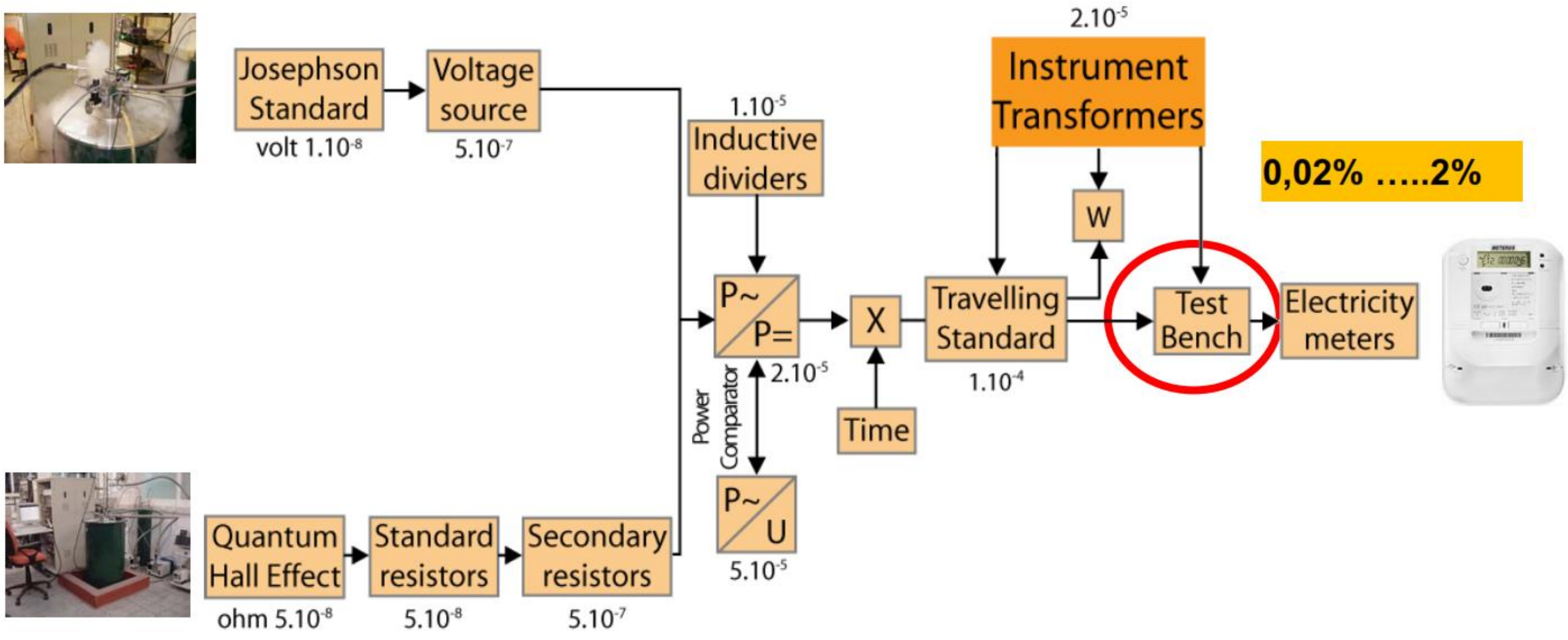




$$P = \frac{1}{N} \sum_{i=1}^{N} u_i \cdot i_i$$

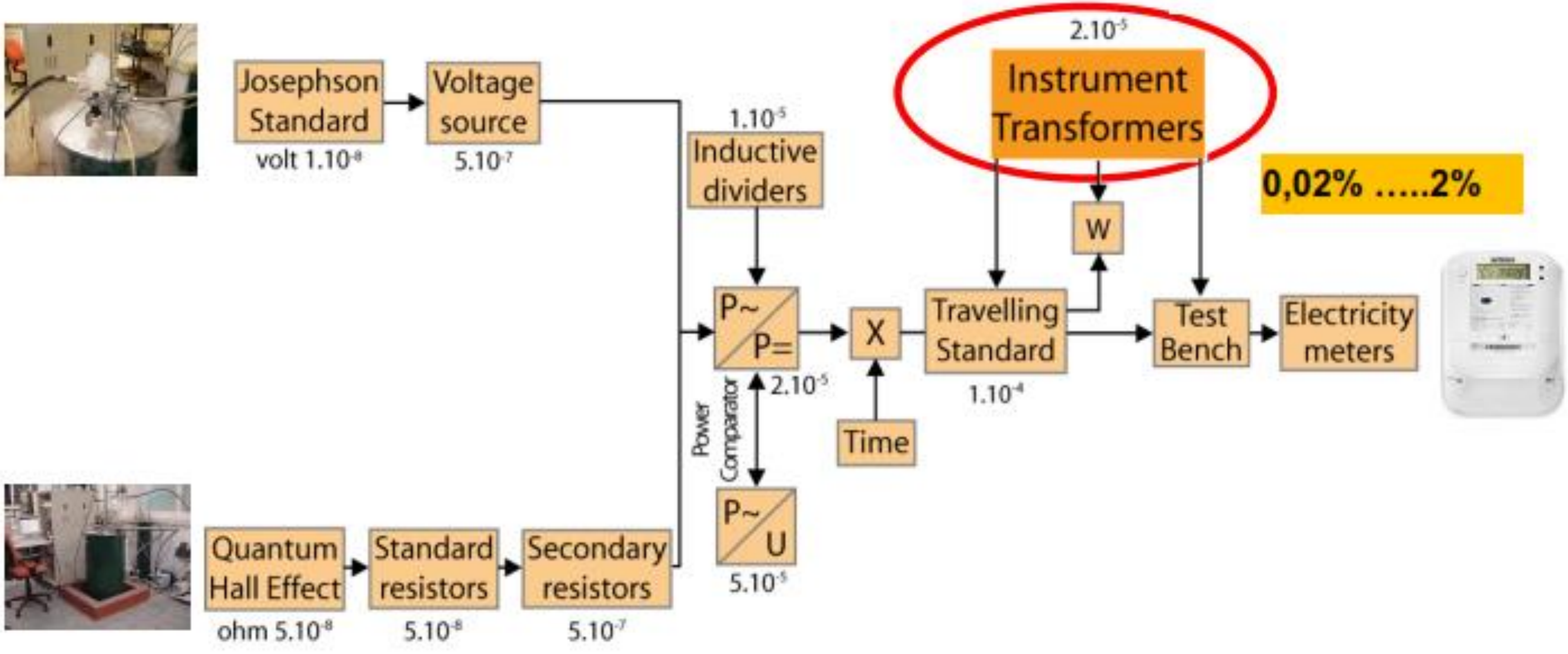
U = 10 µW/VA

Power and energy traceability



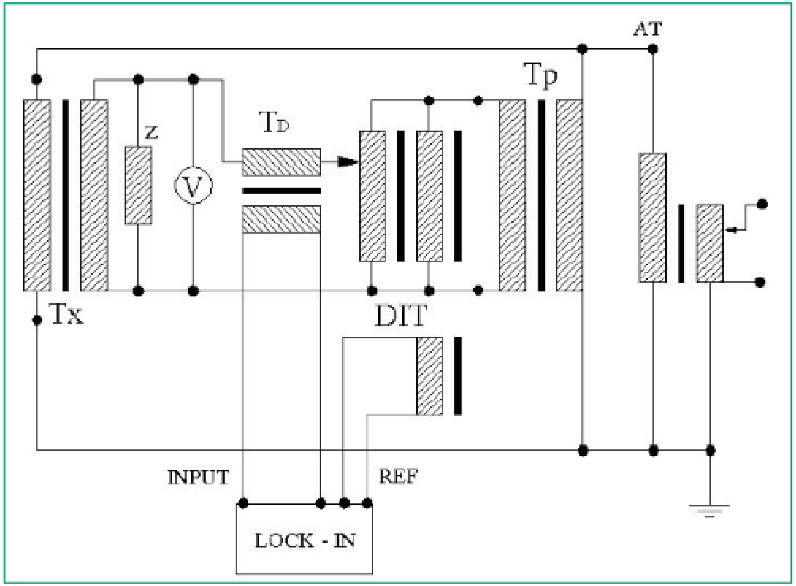
Power and energy traceability

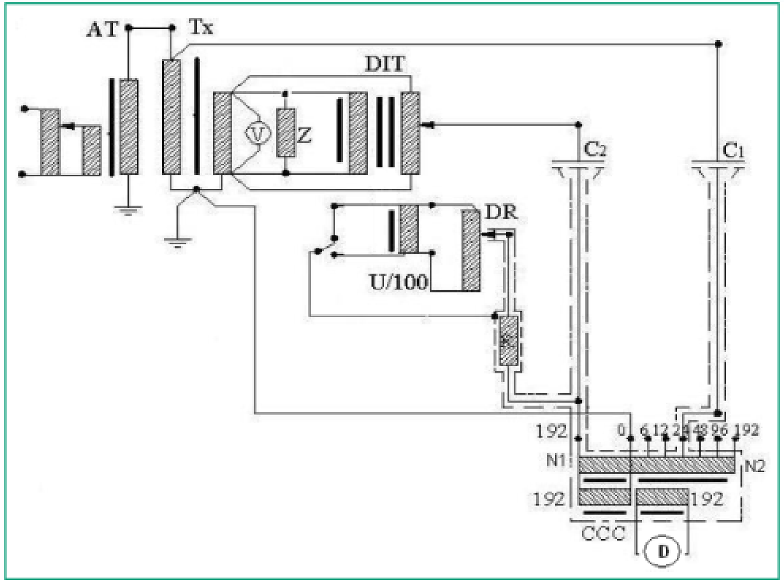
Power and energy traceability



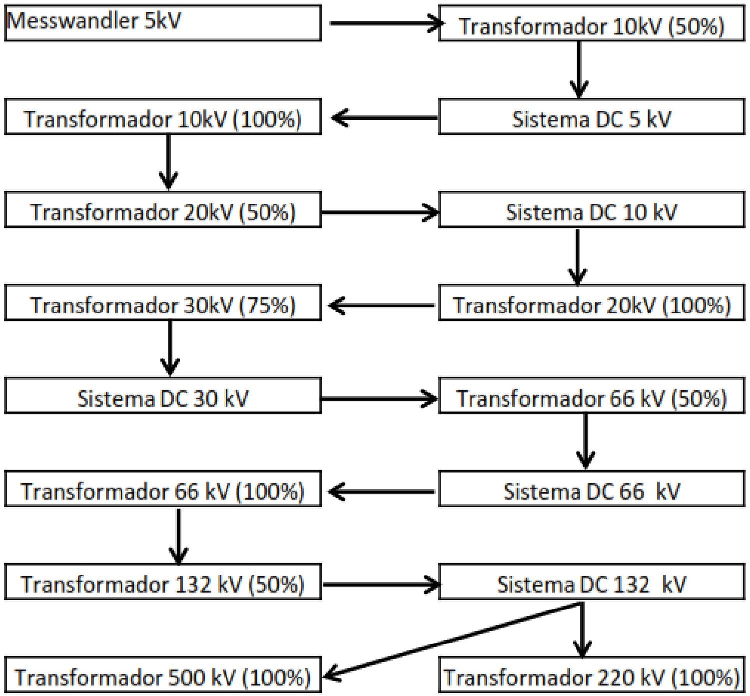
Power and energy traceability

Power and energy traceability

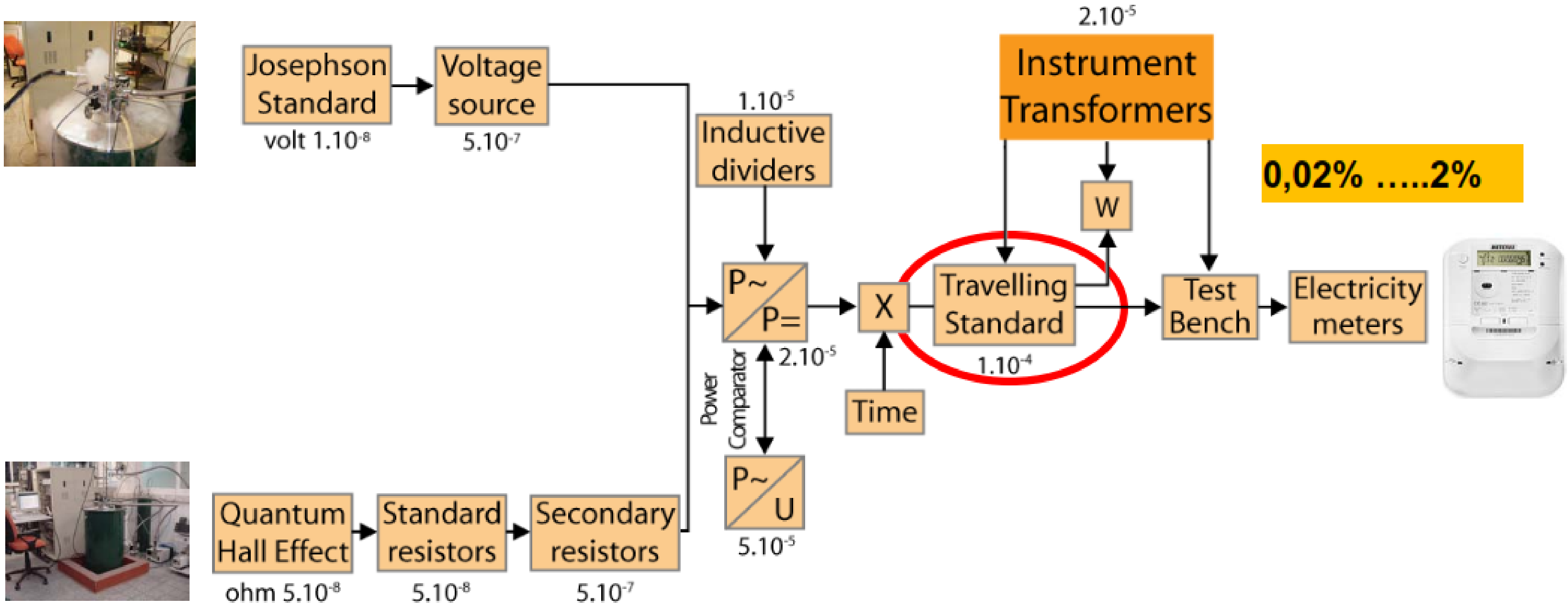




Power and energy traceability



Power and energy traceability



Power and energy traceability

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