SIM Guidelines on the calibration of hydrometers – Cuckow method

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SIM Guidelines on the calibration of hydrometers – Cuckow method

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SIM Guidelines on the calibration of hydrometers – Cuckow method

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PURPOSE

This publication has been discussed within the SIM Metrology Working Group of Mass and Related Quantities (SIM MWG7) with the goal of improving the harmonisation of methods for the calibration of hydrometers within SIM countries.

This document provides guidance to national accreditation bodies to set up minimum requirements for the calibration of hydrometers and gives advice to calibration laboratories to establish practical procedures for the calibration of these kind of instruments.

This document contains detailed examples of the estimation of the uncertainty of measurements.

OFFICIAL LANGUAGE

The English version of this document is considered as the primary reference, however the Spanish version could be used as a reference as close as possible to the English version.

FURTHER INFORMATION

For further information about this publication, contact the member of the SIM MWG7 of the National Metrology Institute of your country.
See http://www.science.oas.org/SIM/default.htm http://www.sim-metrologia.org.br/

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1 PURPOSE OF THE GUIDE

To establish the criteria and requirements that must be met by the procedures, which serve as technical basis for the performance of the calibration of hydrometers, as well as the homologation in the estimate of the uncertainty measurement and evaluation of traceability in them.

Achieving consistency and uniformity in the presentation of uncertainty budgets in the measurements of calibration services, as well as to establish the requirements for the assessment of the calibration services of this type of density meters.

The hydrometers are simple tools but very effective for the measurement of the density of liquids. Hydrometers are widely used in the oil and chemical industries to measure the density or specific gravity of liquids, in the brewing industry to ensure the content of alcohol, in the sugar industry to measure the percentage of sugar present in solutions of sugar cane, and dairy industry to measure the fat content by the density of the milk.

The name of these instruments changes from industry to industry depending on usage. Therefore in the brewing industry they are called the alcoholmeter, in the sugar industry they are called the Brix-hydrometers and lactometers in the dairy industry; the name corresponds to the usage and the units of their graduation, however in general terms they are called hydrometers if they measure the density of liquids.

2 SCOPE OF THE GUIDE

There are essentially two types of hydrometers: hydrometers of constant volume and hydrometers of constant mass. The first ones have a single mark and the possibility of adding mass up to sink the hydrometer to the indicated mark. According to the mass that is added, the density of the liquid is calculated. The second ones have a graduated scale and depending on the density of the liquid the instrument will float and the alignment of the mirror of liquid and the scale will indicate the density of the liquid.

This Technical Guide applies for calibration of hydrometers of constant mass, using the method of Cuckow (hydrostatic weighing) in the measuring range of 600 kg m^{-3} to 2 000 kg m^{-3}, at a temperature value defined by the instrument or at the user requirement, and with expanded uncertainty of calibration (k=2) associated to the scale correction of the instrument up to 0.033 kg m^{-3}.

It is possible to calibrate hydrometers whose scale is graduated in different units than those of the International System (SI); however, the calibration is performed in density and SI units and using the corresponding conversions to calculate the indication errors of the hydrometer in the units of graduation.

The chapter 5.5 of this guide presents the uncertainty values which are recommended as maximum for the calibration of the hydrometers in function of its series and maximum permissible error associated to them; however, it is the responsibility of the calibration laboratory and the client to agree on the appropriated uncertainty value for the calibration of the instrument based on the predetermined use of the instrument and also the cost of the calibration. In the calibration of an instrument, the user should keep in mind that a lower uncertainty value allows a better use of the instrument characteristics; however, the value of uncertainty is related to the cost of this.

This technical guide establishes the minimum requirements for the calibration of hydrometers, to ensure uniformity in the estimation of calibration uncertainty and evaluation of the traceability of the measurement standards of laboratories in the calibration.

It is the responsibility of the calibration laboratory and its client to agree prior to calibration, about the range of density or values of density in which the measuring instrument will be calibrated in accordance to the use of the hydrometer, as well as the cost of the calibration.

The goal of this Guide is to provide general recommendations for the establishment of the calibration procedures and not to present one or more uniform procedures whose use are compulsory.

3 DEFINITIONS

For the purpose of this guide, the following definitions will be applied:

3.1 Adjustment of a measurement system

Set of operations performed on a measurement system to provide prescribed indications, corresponding to given values of the magnitude to measure.

3.2 Nominal amplitude of a nominal interval of indications

Absolute value of the difference between the extreme values of a nominal interval of indications

3.3 Calibration

Operation that, under specified conditions, in a first step, establishes a relation between the quantity values with measurement uncertainties provided by measurement standards and corresponding indications with associated measurement uncertainties and, in a second step, uses this information to establish a relation for obtaining a measurement result form an indication.

3.4 Measuring instrument

Device used for making measurements, alone or in conjunction with one or more supplementary devices.

3.5 Nominal indication interval

Set of quantity values, bounded by rounded of approximate extreme indications, obtainable with a particular setting of the controls of a measuring instrument or measuring system and used to designate that setting.

3.6 Measurand

Quantity intended to be measured

3.7 Measurement Method

Generic description of the logical organization of operations used in a measurement.

3.8 Measurement Model

Mathematical relation among quantities known to be involved in a measurement.

3.9 Measuring System

Set of one or more measuring instruments and often other devices, including any reagent and supply, assembled and adapted to give information used to generate measured quantity values within specified intervals for quantities of specified kinds.

3.10 Metrological Traceability

Property of a measurement result whereby the result can be related to a reference through a documented unbroken chain of calibrations, each contributing to a measurement uncertainty.

3.11 Verification

Provision of objective evidence that a given item fulfils specified requirements.

4 HYDROMETERS SERIES

Hydrometers have characteristics such as resolution, nominal scope, indications interval among others; which should be considered when choosing the appropriate instrument to the needs of use and/or application.

According to the international standard ISO 649/1, there are five main series of hydrometers, which covers a range of 600 kg m^{-3} to 2 000 kg m^{-3}. The production series of these instruments are L20, L50, M50, M100 and S50. The ending 20, 50 and 100 indicates the nominal amplitude of the instrument in kilograms per cubic meter. The ISO 649/1 also includes three sub-series that cover a total range of 600 kg m^{-3} to 1 100 kg m^{-3}, these are the L50SP, M50SP and S50SP.

Each series and sub-series of instruments has associated manufacturing characteristics, in addition to maximum permissible errors associated to each one of them.

The users of hydrometers frequently use them based on the series and the corresponding maximum permissible errors; because of this, it is desirable that the instruments are calibrated and verified (check that their indication errors are less than the maximum permissible), according to the corresponding maximum permissible errors of the ISO 649/1.

One part of the conformity assessment in measuring instruments consists of the evaluation of the following expression:

$$|E \pm U(E)| \leq mpe \tag{4.1}$$

Where the indication error of the instrument E in conjunction with its associated expanded uncertainty $U(E)$ must have a value less than or equal to the maximum permissible error (mpe) corresponding to its series, as shown in Figure 4.1 .

4.1 Maximum permissible errors

In the standard ISO 649/1 [2] the maximum permissible errors are presented for different series of hydrometers. The values of these maximum permissible errors are shown in Table 4.1.

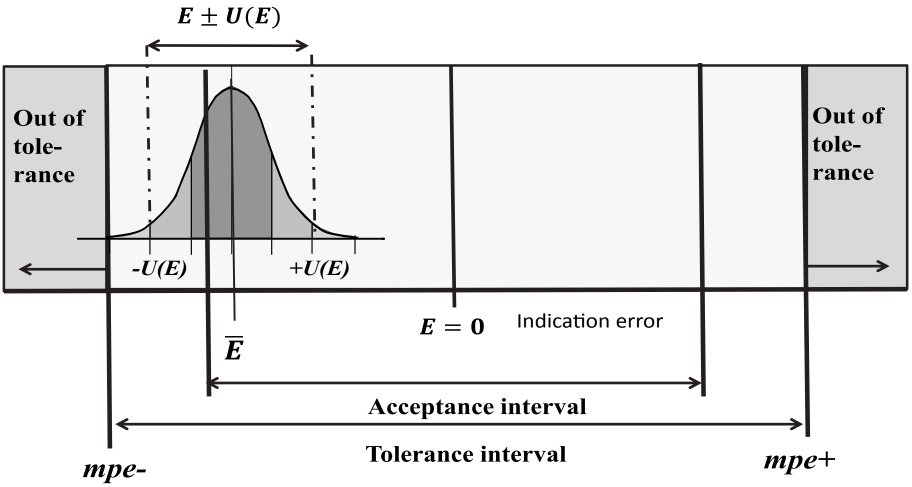

Figure 4.1 Criterion for the evaluation of the conformity of instruments.

Table 4.1. Maximum permissible errors (*mpe*) of hydrometers

Series	*mpe* kg m^{-3}
L20	± 0.2
L50	± 0.5
M50	± 1.0
M100	± 2.0
S50	± 2.0
Sub-series	
L50SP	± 0.3
M50SP	± 0.6
S50SP	± 1.0

4.2 Standard categories of surface tension

Due to the impact that the value of surface tension of the measurement liquid has on the instrument indication, hydrometers are manufactured in three categories [4],

1) Low with surface tension up to 34 mN m^{-1}
2) Medium with surface tension from 35 mN m^{-1} to 55 mN m^{-1}
3) High with surface tension up to 75 mN m^{-1}

5 GENERAL ASPECTS OF THE CALIBRATION

5.1 Measurement system

The calibration laboratory should have everything needed for the handling and calibration of the hydrometers in function of the required uncertainty for the series of instruments that are intended to calibrate. The instruments and accessories needed depend on the particular array that intends the laboratory for the calibration; however the calibration system can include the following equipment:

- Weighing instrument with the possibility of weighing below the instrument
- Liquid of known density and with traceability to the SI
- Instrument to measure the temperature of the liquid (thermometer)
- Instrument to measure the environmental temperature (thermometer)
- Instrument to measure the atmospheric pressure (barometer)
- Instrument to measure the relative humidity of the air (hygrometer)
- Set of weights (mass standards)
- Table conditioned to perform the weighting below the instrument
- Thermostatic bath with temperature control
- Suspension system of the hydrometer (to set up in position of weighing)
- System of visual support for the alignment of the mark to calibrate on the hydrometer with the mirror of the liquid (e. g. magnifying glass or camera).
- Accessories for cleaning and handling of instruments and equipment, e.g. tweezers or hairpin for weights, gloves, knobs, paper among others.

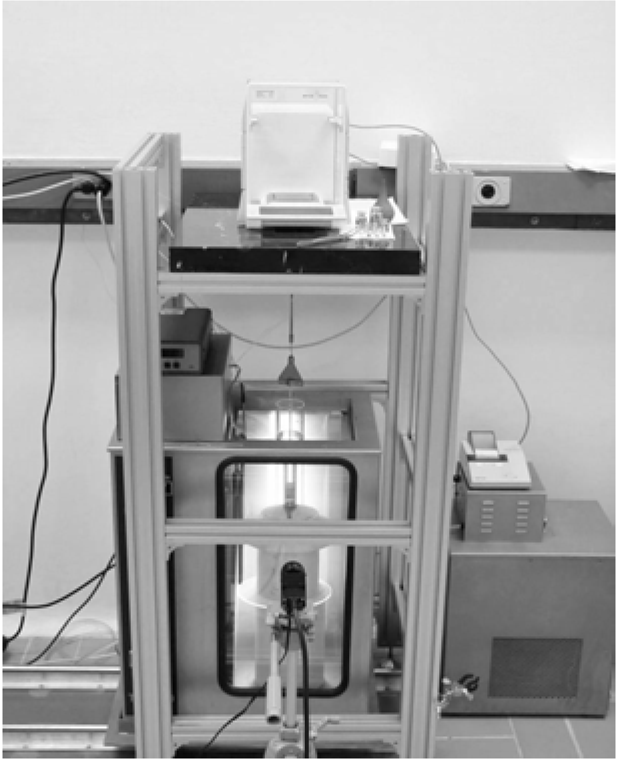

Figure 5.1 System for the calibration of hydrometers by Cuckow method

The suspension system of the hydrometer must be designed in such a manner as to avoid putting on risk the instrument considering that the stem of the hydrometer is very delicate and if there is too much pressure on it, this can break, and on the contrary, if not firmly grasped, the hydrometer can slip and fall.

On the other hand, if the hydrometer is not suspended in the full upright position during the weighing of the instrument partially immersed in the liquid of reference, there will not be reliable readings but errors associated with the alignment of the instrument.

The mentioned instruments (weighting instrument, weights, instruments to measure the environmental conditions and the instrument for measuring the temperature of the liquid), must be calibrated and certified.

Depending on the required uncertainty, it is possible that the laboratory uses the calibrated weighing instrument as measurement standard and does not require the use of weights, which are needed for those laboratories that require the greatest possible accuracy and use the weighing instrument as mass comparator (comparison between the hydrometer and the weights).

The measurement system must be installed in a room free from vibration, air currents and all those factors of influence that could affect the stability of the temperature and/or the indications of the weighing instrument.

It is important that the calibration laboratory controls the temperature gradients both vertical and horizontal (spatial gradients) in the reference liquid in density, as well as the temperature stability before and during the calibration, allowing the hydrometer to acclimatize properly, and to keep the temperature stable during calibration (temporary gradient). The value of the spatial and temporal gradients depends on the required uncertainty in the calibration.

To define the maximum allowable variations in the temperature of the reference liquid (and what type of temperature control is required) should be considered the value of the coefficient of thermal expansion of the reference liquid (α),

$$\Delta\rho = \alpha\Delta t \tag{5.1}$$

Where $\Delta\rho$ is a variation in the density of the liquid and Δt is the variation in temperature; therefore considering that the value of the coefficient of thermal expansion of the water is approx. - 0.000 2 $°C^{-1}$, a variation of 0.2 °C causes a relative variation in the density of water of approximately $– 4 \times 10^{-5}$.

5.2 Preparation of the instrument to calibrate

The metrologist must review the instrument prior to the calibration to ensure that the instrument has a unique and indelible identification (e.g. by Serial No.), that is not broken, that the scale is fixed to the stem of the instrument, and the ballast is kept secure in his position, see figure 5.2

If the instrument is broken, or the scale is not fixed to the stem or the ballast is moved from its position, the instrument cannot be calibrated and should be reported to the user.

Some hydrometers are equipped with a built-in thermometer, this equipment is calibrated with its own procedure of the quantity (temperature), and is not part of the scope of this guide.

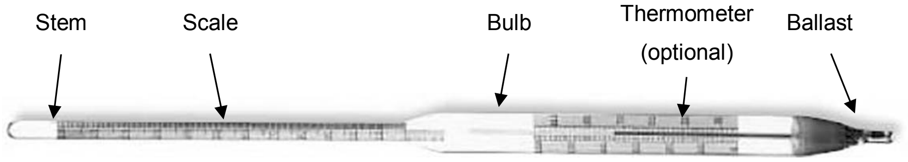

Figure 5.2 Parts of the hydrometers

If the user does not request a different temperature value, the calibration should be performed at 20 °C, however it is important to clarify this issue with the customer because some instruments are designed to be used at 15.56 °C (60 °F) or other temperature value depending on the specific use of the instrument.

Prior to calibration, the hydrometer must be cleaned thoroughly. Before using any solvent, compatibility with the material of the hydrometer must be verified.

Once the instrument is clean, it should be left to acclimatize until it reaches the temperature of measurement. The acclimatization is in the air if the hydrometer weighing is going to be performed in the air or immersed in the liquid (up to the calibration mark) if it is going to perform the weighing of the hydrometer into the fluid. It is very important that the entire measurement system is stable in temperature (acclimatized) to obtain the best results in the calibration.

The indications of the ambient temperature should be registered, also the temperature of the reference liquid, relative humidity and atmospheric pressure, and verify that none of them present significant variations in relation to the required uncertainty of the calibration.

5.3 Density standard (liquid of known density)

The calibration of the hydrometers by Cuckow method requires the use of a liquid whose density is known with the appropriate uncertainty in function of the required uncertainty in the calibration of instruments (see chapter 5.5).

Among the most commonly used liquids as density standard are:

Table 5.1 Liquids commonly used as density standard in the calibration of hydrometers

Liquid	Density @ 20 °C and 101.325 kPa kg m^{-3}	Thermal expansion coeff. @ 20 °C C^{-1}	Surface tension @ 20 °C mN m^{-1}
Water	≈ 998.2	≈ -2×10^{-4}	≈ 73
Pentadecane	≈ 769.0	≈ -9×10^{-4}	≈ 27
n- Nonane	≈ 718.0	≈ -9×10^{-4}	≈ 26

***Note:** Values in the table are placed only as a quick reference, the specific values depend on the particular sample.*

The fluids that are listed in table 5.1 as mentioned above are commonly used as a density standard for the calibration of the hydrometers by Cuckow method, however it is possible to use these liquids or other different as long as the value of density has traceability and their values of surface tension, expansion coefficient and the coefficient of compressibility are known with the required accuracy.

The traceability of the density value of water can be obtained by measuring its temperature, atmospheric pressure and calculating its value by the appropriate formula in regards to the required uncertainty, e.g. Tanaka et al. [13] which allows the greater accuracy for samples of pure water. It is the responsibility of the laboratory to demonstrate the purity of the water and in the same way prove that the formula can be applied (e.g. through measurements of resistivity or conductivity). Similarly it is important to demonstrate the traceability of measurements of temperature and pressure.

As it regards to different liquids other than water, the traceability of its density value in general comes from a previous measurement e.g. hydrostatic weighing using a solid density standard, and therefore the value must be certified with an uncertainty value associated with the value of density according to the required accuracy in the calibration of the hydrometers (see chapter 5.5).

In any case it is important that the calibration laboratory keeps the metrological control of its density standard (reference liquid), and demonstrates the traceability of the density value toward the International System of Units

Because the density value is related to a value of temperature (and pressure, for some cases where the uncertainty requires so), it is very important that the measurements of temperature (and pressure if is the case) are measured with the required accuracy and reliable traceability.

5.4 Nominal values for calibration

For the calibration of hydrometers it is recommended that at least three nominal values are elected to calibrate. The nominal values generally must be distributed in the range of indications of the instrument for calibration, usually the second and penultimate nominal value of the scale are calibrated in addition to the central value.

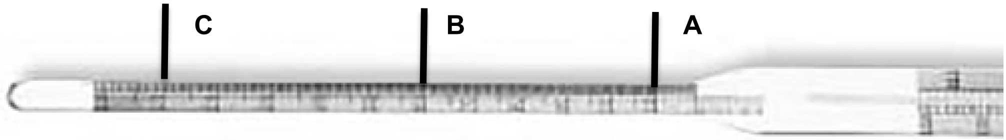

Figure 5.3 Nominal values for the calibration of the hydrometer

In the event that the user requires it, more nominal values of the instrument can be included for the calibration.

A good practice of calibration is to start calibrating the nominal value A (figure 5.3), then the nominal value B and finishing up with the nominal value C, to avoid getting the scale wet above the nominal value to calibrate, which would introduce an error in the measurement by increasing the value of the mass of the hydrometer.

5.5 Required uncertainty in calibration.

It is important that the value of the uncertainty associated with the indication errors obtained during the calibration is consistent with the fabrication characteristics of the hydrometer; due to this the required uncertainty in the calibration of the hydrometers is related to their fabrication series and with their corresponding maximum permissible errors.

The required uncertainty in the calibration of the hydrometers (expanded uncertainty with a coverage factor equal to 2), must be less than or equal to one-third of the mpe (maximum permissible error) of the corresponding series of the instrument under calibration.

$$U_{req}(E) \leq \frac{1}{3}\, mpe \tag{5.2}$$

This relationship between the calibration uncertainty and the maximum permissible error of the instrument allows to obtain a measurement capability index in the calibration of instruments equal to three, $C_m = 3$ [3].

The calibration laboratory should be able to reach the uncertainty value in the calibration of the hydrometers according to the instrument series that are intended to calibrate.

Table 5.2. Required uncertainty in the calibration of the hydrometers depending of the series

Series	*mpe* kg m^{-3}	*Ureq, k=2* kg m^{-3}
L20	± 0.2	≤ 0.067
L50	± 0.5	≤ 0.17
M50	± 1.0	≤ 0.33
M100	± 2.0	≤ 0.67
S50	± 2.0	≤ 0.67
Sub-series		
L50SP	± 0.3	≤ 0.10
M50SP	± 0.6	≤ 0.20
S50SP	± 1.0	≤ 0.33

In table 5.2, it can be seen that for the hydrometers L20 is requires the lowest uncertainty of calibration since they have the lowest maximum permissible error.

5.6 Measurement method for the calibration of hydrometers

The hydrometers calibration by Cuckow method requires the measurement of the hydrometer mass in air, and the measurement of the hydrometer mass partially immersed in the reference liquid up to the mark to calibrate.

The level of the liquid should be aligned with the center of the mark (Figure 5.3). There are different techniques for aligning the mark of the hydrometer to the level of reference liquid as well as different devices to aid the metrologist in this alignment, e.g. camera, zoom lens, etc. [14, 15] The method and device to be used by the metrologist depends on the level of required uncertainty in calibration.

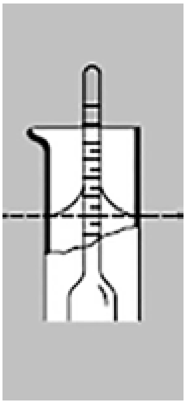

Figure 5.4 Alignment of the hydrometer to the calibration mark

5.6.1 Measurement principle for calibration

The physical principle on which is based the calibration is the Archimedes' principle, which measures the value of buoyancy that the reference liquid has on the suspended hydrometer at the calibration mark. This buoyancy push is measured by using a weighing instrument, through the relationship between the value of the hydrometer mass in the air and the (apparent) mass of the hydrometer partially immersed.

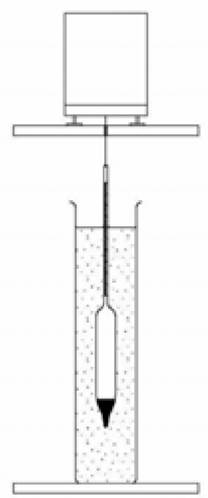

Figure 5.5 Calibration scheme of hydrometers By Cuckow method

5.6.2 Number of required measurements

The number of required measurements (repetitions) for the calibration of the hydrometers will depend on the contributions to the uncertainty that are present due to the weighing process of the hydrometer in relation to the required uncertainty in the calibration of the instrument (see table 5.2).

The metrologist must take into account that in general to increase the number of measurements can help reduce the calibration uncertainty of the instrument, however this can increase the cost of the calibration.

It is not recommended to perform less than three repetitions in mass measurements of the hydrometer for both air measurements and partially immersed in the reference liquid.

5.6.3 General procedure of calibration

The calibration procedure of hydrometers has the following stages:

a. Identification of the characteristics of the instrument, (scope, resolution, series, mpe, and required uncertainty).
b. Cleaning and acclimatization of instrument to calibrate
c. Preparation and stabilization of the measurement system (e.g. reference liquid, weighing instrument, thermostatic bath, thermometer for measuring the temperature of the liquid, instruments for the measurement of environmental conditions, standard weights, among others).
d. Measurement of the hydrometer mass in the air
e. Measurement of the hydrometer mass immersed at the mark to calibrate (calibrated in at least three different nominal values).

6 MEASUREMENT MODEL

The measurement model for the calibration of the hydrometers by Cuckow method is as follows:

The indication error $E(I)$, of the hydrometer is calculated by:

$$E(I) = I(\rho) - \rho_x - \varepsilon_d \tag{6.1}$$

with

$I(\rho)$	Indication of the hydrometer, nominal value of the mark to calibrate; kg m^{-3}
ρ_x	Density at the calibrated mark; kg m^{-3}
ε_d	Error due to the finite resolution of the hydrometer, it has zero mean but contributes to the uncertainty ≈ 0 kg m^{-3}

6.1 Liquid density at the mark level

The value of the liquid density at the calibrated mark (ρ_x), is calculated as follows,

$$\rho_x = (\rho_L f_{t,L} - \rho_a f_{t,a}) \cdot \left[\frac{m_a + \frac{\pi D \gamma_x}{g}}{m_a - m_L + \frac{\pi D \gamma_L}{g}}\right] + \rho_a f_{t,a} \qquad (6.2)$$

Where:

ρ_L	Density of the reference liquid at the measurement conditions; kg m^{-3}
ρ_a	Density of the air during the measurement; kg m^{-3}
$f_{t,x}$	Correction factor of volume of the hydrometer material due to a change in temperature Δt; the second sub index indicates whether this change relates to the air temperature or to the liquid; dimensionless.
m_a	Apparent mass of the hydrometer in the air; kg
m_L	Apparent mass of the hydrometer immersed until the calibration mark; kg
D	Diameter of the stem to the level of the mark to calibrate; m
g	Acceleration due to local gravity; m s^{-2}
γ_L	Surface tension of the reference liquid; N m^{-1}
γ_x	Surface tension of the liquid where the hydrometer will be used; N m^{-1}

6.2 Density of reference liquid

If water is used with the necessary characteristics to be used as reference material in density, the value of density of this water is calculated by measuring the temperature and pressure (depending on the required uncertainty), and these values of temperature and pressure are entered in the corresponding formula. The formula that allows the least uncertainty in the calculation of water density in the range of 0 °C to 40° C is the formula of Tanaka [13], however there are some other formulae that could be used if the required uncertainty in the calibration of the hydrometer allows it, see Annex B.

The value of density of the reference liquid can be obtained by a measurement and prior certification. This density value must be referred to a temperature value and reference pressure, therefore the density must be calculated at temperature t_x and pressure p_x of measurement.

The liquid density at measurement conditions (for the calibration of the hydrometer) is calculated with the following expression,

$$\rho_L = \rho_{Cert}\ {f_t}^{-1}\ {f_p}^{-1} - \varepsilon_{est} \tag{6.3}$$

Where,

ρ_L	Density at a temperature t_x and pressure p_x; kg m^{-3}
ρ_{Cert}	Density of the liquid at reference conditions, at temperature T and pressure P; kg m^{-3}
f_p	Correction factor of density of the reference material due to a change of pressure Δp; dimensionless.
ε_{est}	Error of density due to the (lack of) stability of the value of reference density; kg m^{-3}

6.3 Calculation of the correction factor of the volume / density due to a change in temperature

The correction factor to correct the value of volume or density of reference due to a change in temperature $(\Delta t = t_x - t_{Ref})$, is calculated with the following expression,

$$f_t = 1 + \alpha(t_x - t_{Ref}) \tag{6.4}$$

Where,

f_t	Temperature correction factor, dimensionless
α	Volumetric expansion coefficient of the material, °C^{-1}
t_x	Temperature x, to which it is desirable to know the volume / material density, °C
t_{Ref}	Reference temperature of the density of the fluid, °C

Note: *The volumetric expansion coefficient of the fluid α_{vol} can be expressed in different ways: in other units (e.g., in kg m^{-3} °C^{-1}) or as a function of the temperature, (e.g., as a polynomial $\alpha(t) = a_0 + a_1 t + \ldots + a_n t^n$). In these cases, the mathematical model of the factor f_t might be different from the one proposed.*

6.4 Calculation of the correction factor of density due to a change in the pressure value

The correction factor to correct the density of reference due to a change in pressure ($\Delta p = p_x - p_{Ref}$), is calculated with the following expression:

$$f_p = 1 - \beta(p_x - p_{Ref}) \tag{6.5}$$

Where,

f_p	Correction factor by pressure, dimensionless
β	Isothermal compressibility coefficient of the fluid, Pa^{-1}
p_x	Pressure x, to which it is desirable to know the fluid density, Pa
p_{Ref}	Pressure of reference to the known density of the fluid, Pa

***Note:** The isothermal compressibility coefficient of the fluid β can be expressed in different ways: in other units (e.g, in kg m^{-3} Pa^{-1}) or as a function of pressure, (e.g, as a polynomial $\beta(p) = b_0 + b_1 p + \ldots + b_n p^n$). In these cases, the mathematical model of the factor f_p might be different from the one proposed.*

6.5 Air density

The air density and its uncertainty is calculated by entering the values of air temperature, relative humidity and atmospheric pressure, in the corresponding formula.

The formula that high accuracy offers is known as CIPM 2007 [17]; however, if the required uncertainty in the calibration of the hydrometers allows it, other formulas can be used to calculate the density of the air, which are approximations that although have greater simplicity for calculation the air density, have higher associated uncertainty than the formula CIPM 2007, see annex B.

6.6 Apparent mass of the hydrometer in air or partially immersed in the liquid

The measurement of apparent mass of the hydrometer in the air or partially immersed (up to the mark to calibrate), can be performed by either direct reading, using the weighing instrument as measurement standard [19], or by using the weighing instrument and standard weights using a comparison method.

If the apparent mass of the hydrometer (into the air or liquid) is obtained by direct reading of the weighing instrument, this measurement is performed as the difference of two indications, the indication of the weighing instrument with load (with the hydrometer and the suspension, if this is the case) and the weighing instrument without load (or only with the suspension). The expression to get the apparent mass of the hydrometer is the following,

$$m_{a,L} = (R_{hyd} - \varepsilon_I)\left(1 - \frac{\rho_a}{\rho_c}\right) - \varepsilon_{dig\ hyd} - (R_0 - \varepsilon_{dig0}) \tag{6.6}$$

R_{hyd}	Average of readings of the weighing instrument, kg
ε_I	Error of indication of the calibrated weighing instrument, kg
ρ_c	Conventional density of the standard weights, ≈8 000 kg m^{-3}
R_0	Indication of the weighing instrument without charge, if the weighing instrument is tared before placing the load, it is equal to 0 kg
$\varepsilon_{dig\ hyd}$	Error due to the finite resolution of the weighing instrument with the hydrometer suspended or supported in the dish for weighing, it has zero mean but contributes to the uncertainty, ≈0 kg
ε_{dig0}	Error due to the finite resolution of the weighing instrument without load, it has zero mean but contributes to the uncertainty, ≈0 kg

If the apparent mass of the hydrometer (into the air or liquid) is obtained by comparison against standard weights, the expression is as follows,

$$m_{a,L} = [m_s + (R_{hyd} - R_{ms})]\left(1 - \frac{\rho_a}{\rho_c}\right) - \varepsilon_{dig\ hyd} - \varepsilon_{dig\ ms} \tag{6.7}$$

Where,

m_s	Certified mass of the standard weights that balance the indication of the hydrometer, kg
R_{hid}	Indication of the weighing instrument with the hydrometer (and the suspension if applicable) suspended from the weighing instrument, kg
R_{ms}	Indication of the weighing instrument with the standard weights (and the suspension if applicable) placed in the dish of the weighing instrument, kg
$\varepsilon_{dig\ ms}$	Error due to the finite resolution of the weighing instrument with the weights placed on the weighing dish (and the suspension if applicable, ≈0 kg

Note: *In any case, the mass of the suspension of the hydrometer must be removed from the value of the mass of the hydrometer in the air or from the value of mass of hydrometer partially immersed in the liquid of reference.*

It is possible, based on the liquid density and the value of nominal density of the hydrometer to calibrate, to require the use of a ballast to immerse the hydrometer until the mark to calibrate, for this reason it is necessary to measure the mass of the ballast immersed in the fluid of reference and subtract this mass from the hydrometer mass in the liquid.

6.7 Surface tension of the liquid of reference

The value of the surface tension of the liquid of reference must be measured or known with sufficient accuracy in view of the required accuracy in the calibration. In some fluids, the value of the surface tension changes in accordance with pollution of the surface layer of the liquid (e.g., water) and, for this reason for such fluids, it is recommended that you renew the surface of the liquid continuously to maintain stable value of surface tension.

6.8 Surface tension of the liquid where the hydrometer will be used

Hydrometers must be calibrated for the value of surface tension of liquids where normally the calibrated instrument will be used, this allows the user to have a minor error of indication due to this factor.

In the international standard ISO 649/1 [2], three categories are presented for the surface tension of the hydrometers, as well as characteristic surface-tension values to use depending on these categories and the nominal value of density of the instrument.

When the metrologist does not have an indication of the value of surface tension of the liquid to measure (by the user in use of the instrument), a surface tension value can be selected as a function of its category and the nominal value of the density to calibrate.

7 UNCERTAINTY OF MEASUREMENT

The calculation of the calibration uncertainty of the hydrometer is carried out by applying the law of propagation of uncertainties to the mathematical model that describes the calibration [10].

In general, the combined standard uncertainty $u_c(y)$ is the positive root of the square root of the combined variance $u_c^2(y)$, which is obtained by

$$u_c^2(y) = \sum_{i=1}^{N} \left(\frac{\delta f}{\delta x_i}\right)^2 u^2(x_i) \qquad (7.1)$$

The basic formula for the calibration of each of the nominal values under test is (6.1), with the following variances,

$$u^2\big(E(I)\big) = u^2(I) + u^2(\rho_x) + u^2(\varepsilon_d) \tag{7.2}$$

Note: *The variance is the value of the standard deviation squared, applied to the concepts of uncertainty, the standard uncertainty squared is a variance.*

7.1 Uncertainty associated to the indication of the hydrometer during the weighing in the liquid

The uncertainty due to the hydrometer indication in the calibration is associated with the repeatability of the measurements. This repeatability can be significantly improved with the support of optical devices, e.g. cameras or magnifying lenses.

$$u(I) = \frac{s(I)}{\sqrt{n}}; \qquad u(I) = \frac{1}{\sqrt{n}}\sqrt{\frac{\sum(I_i - \bar{I})^2}{n-1}} \tag{7.3}$$

With,

s Standard deviation of the measurements; kg m^{-3}
n Number of repetitions

7.2 Uncertainty associated to the resolution of the hydrometer

The uncertainty due to the resolution of the hydrometer is estimated by assigning a rectangular probability distribution to the resolution of the instrument. The resolution of the hydrometer is estimated by the metrologist as the minimum perceptible variation of the instrument indication, and which could be reduced significantly with the use of auxiliary optical devices,

$$u(\varepsilon_d) = \frac{d}{\sqrt{12}} \tag{7.4}$$

With,

d Resolution assigned to the hydrometer, kg m^{-3}

7.3 Uncertainty associated to density at the mark level of calibration

The uncertainty due to the measured density at the level of the calibrated mark is obtained from the model (6.2).

The combined variance (uncertainty squared) of density at the level of the mark to calibrate is as follows,

$$\begin{aligned} u^2(\rho_x) = {} & \left(\frac{\delta\rho_x}{\delta\rho_L}\right)^2 u^2(\rho_L) + \left(\frac{\delta\rho_x}{\delta\rho_a}\right)^2 u^2(\rho_a) + \left(\frac{\delta\rho_x}{\delta m_a}\right)^2 u^2(m_a) + \left(\frac{\delta\rho_x}{\delta m_L}\right)^2 u^2(m_L) \\ & + \left(\frac{\delta\rho_x}{\delta D}\right)^2 u^2(D) + \left(\frac{\delta\rho_x}{\delta g}\right)^2 u^2(g) + \left(\frac{\delta\rho_x}{\delta \gamma_L}\right)^2 u^2(\gamma_L) + \left(\frac{\delta\rho_x}{\delta f_{t,L}}\right)^2 u^2(f_{t,L}) \\ & + \left(\frac{\delta\rho_x}{\delta f_{t,a}}\right)^2 u^2(f_{t,a}) \end{aligned} \tag{7.5}$$

The sensitivity coefficients are the following,

$$\frac{\delta\rho_x}{\delta\rho_L} = f_{t,L} \cdot \left[\frac{m_a + \frac{\pi D\gamma_x}{g}}{m_a - m_L + \frac{\pi D\gamma_L}{g}}\right] \tag{7.6}$$

$$\frac{\delta\rho_x}{\delta\rho_a} = f_{t,a} - f_{t,a} \cdot \left[\frac{m_a + \frac{\pi D\gamma_x}{g}}{m_a - m_L + \frac{\pi D\gamma_L}{g}}\right] \tag{7.7}$$

$$\begin{aligned} \frac{\delta\rho_x}{\delta m_a} = {} & \left\{\left[\left(m_a - m_L + \frac{\pi D\gamma_L}{g}\right) \cdot (\rho_L f_{t,L} - \rho_a f_{t,a})\right] - \left[(\rho_L f_{t,L} - \rho_a f_{t,a}) \cdot \left(m_a + \frac{\pi D\gamma_x}{g}\right)\right]\right\} \\ & \cdot \left(m_a - m_L + \frac{\pi D\gamma_L}{g}\right)^{-2} \end{aligned} \tag{7.8}$$

$$\frac{\delta\rho_x}{\delta m_L} = \left\{\left[(\rho_L f_{t,L} - \rho_a f_{t,a}) \cdot \left(m_a + \frac{\pi D\gamma_x}{g}\right)\right]\right\} \cdot \left(m_a - m_L + \frac{\pi D\gamma_L}{g}\right)^{-2} \tag{7.9}$$

$$\begin{aligned} \frac{\delta\rho_x}{\delta D} = {} & \left\{\left[\left(m_a - m_L + \frac{\pi D\gamma_L}{g}\right) \cdot (\rho_L f_{t,L} - \rho_a f_{t,a}) \left(\frac{\pi D\gamma_x}{g}\right)\right]\right. \\ & \left. - \left[(\rho_L f_{t,L} - \rho_a f_{t,a}) \cdot \left(m_a + \frac{\pi D\gamma_x}{g}\right) \cdot \left(\frac{\pi D\gamma_L}{g}\right)\right]\right\} \cdot \left(m_a - m_L + \frac{\pi D\gamma_L}{g}\right)^{-2} \end{aligned} \tag{7.10}$$

$$\frac{\delta\rho_x}{\delta g} = \left\{\left[\left(m_a - m_L + \frac{\pi D\gamma_L}{g}\right)\cdot(\rho_L f_{t,L} - \rho_a f_{t,a})\cdot\left(-\frac{\pi D\gamma_x}{g^2}\right)\right] - \left[(\rho_L f_{t,L} - \rho_a f_{t,a})\cdot\left(m_a + \frac{\pi D\gamma_x}{g}\right)\cdot\left(-\frac{\pi D\gamma_L}{g^2}\right)\right]\right\}\cdot\left(m_a - m_L + \frac{\pi D\gamma_L}{g}\right)^{-2} \tag{7.11}$$

$$\frac{\delta\rho_x}{\delta\gamma_L} = \left[(\rho_L f_{t,L} - \rho_a f_{t,a})\cdot\left(m_a + \frac{\pi D\gamma_x}{g}\right)\cdot\left(\frac{\pi D}{g}\right)\right]\cdot\left(m_a - m_L + \frac{\pi D\gamma_L}{g}\right)^{-2} \tag{7.12}$$

$$\frac{\delta\rho_x}{\delta f_{t,L}} = \rho_L\left[\frac{m_a + \frac{\pi D\gamma_x}{g}}{m_a - m_L + \frac{\pi D\gamma_L}{g}}\right] \tag{7.13}$$

$$\frac{\delta\rho_x}{\delta f_{t,a}} = \rho_a - \rho_a\left[\frac{m_a + \frac{\pi D\gamma_x}{g}}{m_a - m_L + \frac{\pi D\gamma_L}{g}}\right] \tag{7.14}$$

7.3.1 Uncertainty associated to the correction factor of volume / density of the hydrometer, due to a difference in temperature between the air/liquid and the reference temperature

If there is a temperature difference between the value of the reference temperature and the value of temperature of measurement, the variance associated with this correction factor is the following:

$$u^2\left(f_{t\,a,L}\right) = \alpha^2 u^2\left(t_{a,L}\right) + \left(t_{a,L} - t_{Ref}\right)^2 u^2(\alpha) \tag{7.15}$$

With

$u\left(t_{a,L}\right)$	Uncertainty of the temperature of air or liquid
$u(\alpha)$	Uncertainty of the coefficient of volumetric expansion of the hydrometer material

7.3.2 Uncertainty associated to density of the liquid of reference

If water was used as reference liquid in density, the uncertainty of the value of water density at the temperature (and if applicable the pressure) of measurement is estimated according to the formula used, including the contributions due to temperature, pressure and the formula itself.

The uncertainty of temperature (and if applicable the pressure), you should consider: the uncertainty of calibration of the instrument, the contribution due to the resolution, the stability of the parameter under measurement in the fluid, gradients, etc. The variance of the temperature would be calculated as follows.

$$u^2(t) = u^2(\varepsilon_t) + \left(\frac{d_t}{\sqrt{12}}\right)^2 + \left(\frac{s(t)}{\sqrt{n}}\right)^2 + \left(\frac{t_{Max}-t_{min}}{\sqrt{12}}\right)^2 \tag{7.16}$$

$u(\varepsilon_t)$	Uncertainty of the calibration error of the thermometer; °C
d_t	Thermometer resolution; °C
$s(t)$	Standard deviation of the temperature measurements; °C
t_{Max}	Maximum value of temperature, °C
t_{min}	Minimum temperature value, °C

If the density value of the liquid of reference comes from a previous calibration, the density value and uncertainty must be certified to certain conditions of temperature and pressure, so that by applying the law of propagation of uncertainties to the model (6.3), the variance of the value of density of reference is obtained as,

$$\begin{aligned} u^2(\rho_L) = {} & \left[f_t^{-1} f_p^{-1} u(\rho_{cert})\right]^2 + \left[\rho_L f_t^{-2} f_p^{-1}\right]^2 \left\{[\alpha_L u(t)]^2 + [(t_x - t_{Ref}) u(\alpha_L)]^2\right\} \\ & + [\rho_L f_t^{-1} f_p^{-2}]^2 \left\{[-\beta u(p)]^2 + [(p_{Ref} - p_x) u(\beta)]^2\right\} + u^2(\varepsilon_{est}) \end{aligned} \tag{7.17}$$

The standard uncertainty associated with the error due to the stability of density can be estimated considering a maximum value of variation (D_{drift}), and consider this maximum value as a medium interval of uniform probability.

$$u(\varepsilon_{est}) = \frac{D_{drift}}{\sqrt{3}} \tag{7.18}$$

Note: the value of maximum variation is considered as a medium range.

7.3.3 Uncertainty associated to air density

The uncertainty of the air density is estimated based on the formula that is used to calculate the value,

$$u^2(\rho_a) = \left[\frac{\delta\rho_a}{\delta t_a} u(t_a)\right]^2 + \left[\frac{\delta\rho_a}{\delta p} u(p)\right]^2 + \left[\frac{\delta\rho_a}{\delta hr} u(hr)\right]^2 + u_{form}^2 \tag{7.19}$$

With,

$\frac{\delta\rho_a}{\delta t_a}$	Sensitivity coefficient of the air density with regard to the temperature, kg m^{-3} C^{-1}

$\frac{\delta \rho_a}{\delta p}$	Sensitivity coefficient of the air density with regard to the pressure, kg m^{-3} Pa^{-1}
$\frac{\delta \rho_a}{\delta hr}$	Sensitivity coefficient of the air density with regard to the relative humidity of the air, kg m^{-3}
$u(t_a)$	Standard uncertainty of the temperature, °C
$u(p)$	Standard uncertainty of the pressure, Pa
$u(hr)$	Standard uncertainty of the relative humidity of the air, %
u_{form}^2	Uncertainty associated with the formula used to calculate the density of the air, kg m^{-3}

The uncertainties due to the temperature, pressure and relative humidity must contain at least the contributions due to the calibration of the measuring instrument, the resolution and the gradients in their case.

7.3.4 Uncertainty associated to the apparent mass of the hydrometer in the air or partially immersed up to the mark to calibrate.

The variance associated with the apparent mass of the hydrometer in the air or partially immersed up to the mark to calibrate is estimated depending on the method of measurement, if the measurement is performed directly the uncertainty is derived from the formula (6.6) as:

$$u^2(m_{a,L}) = \left[\left(1-\frac{\rho_a}{\rho_c}\right)\cdot\left(\frac{s(R)}{\sqrt{n}}\right)\right]^2 + \left[\left(1-\frac{\rho_a}{\rho_c}\right)\cdot u(\varepsilon_I)\right]^2 + \left[\left(\frac{(R_{hid}-\varepsilon_I)}{\rho_c}\right)\cdot u(\rho_a)\right]^2 + \left(\frac{d_b}{\sqrt{6}}\right)^2 \quad (7.20)$$

If the apparent mass was obtained by comparison against mass standards (weights), formula (6.7), the variance is,

$$u^2(m_{a,L}) = \left[\left(1-\frac{\rho_a}{\rho_c}\right)\cdot\left(\frac{s(\Delta R)}{\sqrt{n}}\right)\right]^2 + \left[\left(1-\frac{\rho_a}{\rho_c}\right)\cdot u(m_s)\right]^2 + \left[\left(\frac{m_s}{\rho_c}\right)\cdot u(\rho_a)\right]^2 + \left(\frac{d_b}{\sqrt{6}}\right)^2 \quad (7.21)$$

with

$s(R)$	Standard deviation of the indications of the weighing instrument weighing the hydrometer; kg
$s(\Delta R)$	Standard deviation of the indications of the weighing instrument when the hydrometer is weighed and when standard weights are placed, kg
d_b	Resolution of the weighing instrument; kg

Note: The uncertainty due to the finite resolution of the weighing instrument, is calculated as:

$$u^2(\varepsilon_{dig\ x}) = \left(\frac{d_b}{\sqrt{12}}\right)^2 \tag{7.22}$$

As the resolution contributes on two occasions because the measurement of mass is in general a difference between two indications, the indication of the weighing instrument with the hydrometer and the weighing instrument without charge, or the weighing instrument with the hydrometer and the weighing instrument with the standard weights, the variance of this double contribution is:

$$2 \cdot \left(\frac{d_b}{\sqrt{12}}\right)^2 = \left(\frac{d_b}{\sqrt{6}}\right)^2 \tag{7.23}$$

7.3.5 Uncertainty associated to the stem diameter at the level of the mark to calibrate

The variance associated with the diameter of the stem depends on the method and instrument used to measure it. For example, if the diameter of the stem is measured with a vernier, the uncertainty of the diameter should at least contain the contributions due to the calibration of the instrument, the resolution and the variability of the diameter.

$$u^2(D) = u^2(\varepsilon_V) + \left(\frac{d_V}{\sqrt{12}}\right)^2 + \left(\frac{s(D)}{\sqrt{n}}\right)^2 \tag{7.24}$$

where,

$u(\varepsilon_V)$	Standard uncertainty of the calibration of the vernier, m
d_V	Resolution of the vernier, m
$s(D)$	Standard deviation of the measurements of the diameter, m

For some cases it is possible to estimate the uncertainty of the diameter by estimating a maximum and minimum value of the diameter of the stem,

$$u(D) = \frac{D_{Max} - D_{min}}{\sqrt{12}} \tag{7.25}$$

where,

D_{Max}	Maximum diameter of the stem at the level of the mark to calibrate, m
D_{min}	Minimum diameter of the stem at the level of the mark to calibrate, m

7.3.6 Uncertainty associated to the value of local gravity

The uncertainty associated with the value of the local gravity can be estimated through a maximum and minimum value of the local gravity,

$$u(g) = \frac{g_{Max} - g_{min}}{\sqrt{12}} \tag{7.26}$$

with,

g_{Max}	Maximum value of gravity, m s^{-2}
g_{min}	Minimum value of gravity, m s^{-2}

7.3.7 Uncertainty associated to the value of surface tension of the liquid of reference

The uncertainty of surface tension of the liquid of reference may come from a measurement, in which case this measurement of surface tension must correspond to the temperature at the measuring temperature, considering that the surface tension value depends on the value of temperature.

When there is an estimate of the value of surface tension, the uncertainty must be estimated through a maximum and minimum value of the surface tension, taking care to ensure that these values actually represent these limits,

$$u(\gamma_L) = \frac{\gamma_{L\,max} - \gamma_{L\,min}}{\sqrt{12}} \tag{7.27}$$

with,

$\gamma_{L\,max}$	Maximum value of surface tension of the liquid of reference, N m^{-1}
$\gamma_{L\,min}$	Minimum value of surface tension of the liquid of reference, N m^{-1}

7.4 Expanded uncertainty of the indication error of the hydrometer

From the combined standard uncertainty of the indication error, formula (7.2), it is estimated the expanded uncertainty with (7.28), where k, is usually taken as equal to 2, to obtain a level of confidence of approximately equal to 95 per cent (based on the central limit theorem).

$$U(E(I)) = k \cdot u(E(I)) \tag{7.28}$$

Note: *For more information on the coverage factor k, please refer to annex C*

7.5 Table of uncertainty budget

With the intention to maintain the information orderly, it is convenient to fill the uncertainties budget table.

Due to large number of variables are involved, it is possible to work with multiple tables of uncertainty budget, depending on the sources of information.

Table 7.1 Table of uncertainty budget for the error of indication

	Source	Value[1]	Variability[2]	Distribution[3]	Standard uncertainty	Sensitivity Coefficient	Contribution
1	Hydrometer indication	I			$u(I)$	1	$u(I)$
2	Density to the mark	ρ_x			$u(\rho_x)$	1	$u(\rho_x)$
3	Hydrometer resolution	ε_d			$u(\varepsilon_d)$	1	$u(\varepsilon_d)$
						Combined standard uncertainty	$u\big(E(I)\big)$
4	**Indication error**[4]	$E(I)$				Expanded uncertainty, k=2	$U\big(E(I)\big)$

---

[1] Record the best estimate of the magnitude of entry

[2] Record the value of the variability of the standard uncertainty associated.

[3] Record the type of distribution associated with the source of uncertainty

[4] Record the best estimate of the error of indication during calibration.

Table 7.2 Table of uncertainty budget for the value of density to the level of the mark

	Source	Value	Variability	Distribution	Standard Uncertainty	Sensibility Coefficient	Contribution
1	Density of the reference liquid	ρ_L			$u(\rho_L)$	$\frac{\delta\rho_x}{\delta\rho_L}$	$\frac{\delta\rho_x}{\delta\rho_L}u(\rho_L)$
2	Density of air	ρ_a			$u(\rho_a)$	$\frac{\delta\rho_x}{\delta\rho_a}$	$\frac{\delta\rho_x}{\delta\rho_a}u(\rho_a)$
3	Apparent mass of the hydrometer in the air	m_a			$u(m_a)$	$\frac{\delta\rho_x}{\delta m_a}$	$\frac{\delta\rho_x}{\delta m_a}u(m_a)$
4	Apparent mass of the hydrometer in the fluid	m_L			$u(m_L)$	$\frac{\delta\rho_x}{\delta m_L}$	$\frac{\delta\rho_x}{\delta m_L}u(m_L)$
5	Correction factor of air temperature	$f_{t,a}$			$u\left(f_{t,a}\right)$	$\frac{\delta\rho_x}{\delta f_{t,a}}$	$\frac{\delta\rho_x}{\delta f_{t,a}}u\left(f_{t,a}\right)$
6	Correction factor of liquid temperature	$f_{t,L}$			$u\left(f_{t,L}\right)$	$\frac{\delta\rho_x}{\delta f_{t,L}}$	$\frac{\delta\rho_x}{\delta f_{t,L}}u\left(f_{t,L}\right)$
7	Surface tension of the liquid of reference	γ_L			$u(\gamma_L)$	$\frac{\delta\rho_x}{\delta\gamma_L}$	$\frac{\delta\rho_x}{\delta\gamma_L}u(\gamma_L)$
8	Surface tension of liquid x	γ_x			---	---	---
9	Stem diameter	D			$u(D)$	$\frac{\delta\rho_x}{\delta D}$	$\frac{\delta\rho_x}{\delta D}u(D)$
10	Local gravity acceleration	g			$u(g)$	$\frac{\delta\rho_x}{\delta g}$	$\frac{\delta\rho_x}{\delta g}u(g)$
						Combined standard uncertainty	$u(\rho_x)$
11	**Density at level mark**	ρ_x				Expanded uncertainty,, k=2	$U(\rho_x)$

8 CONTENT OF THE CALIBRATION CERTIFICATE

This section contains advice on the information that can be useful to provide in a calibration certificate. It is intended to be consistent with the requirements of the ISO/IEC 17025, which have priority.

8.1 General Information

- Identification of the calibration laboratory,
- Reference to the accreditation (accreditation entity, number of accreditation),
- Identification of the certificate (calibration number, issue date, number of pages),
- Signature(s) of authorized person(s),
- Customer Identification,
- Identification of the calibrated instrument,
- Information of the instrument identification (manufacturer, serial number, scale division),
- Warning that the certificate could be reproduced only in full unless the calibration laboratory authorized the opposite in writing.

8.2 Information about the calibration procedure

- Date of measurements,
- Place of calibration,
- Environmental conditions during calibration,
- Information about the instrument (Surface tension of the liquid where it will be used), to which it was calibrated,
- Reference and/or description of the applied procedure,
- Agreements with the client e.g. the scope of the agreed calibration, requested uncertainty, surface tension of the liquid where it will be used, metrological specifications for which has been declared conformity.
- Information on the traceability of the measurement results.

8.3 Measurement results

- The indications (nominal values) and the errors for density values of applied test or related errors to the indications – as discrete values.
- The expanded uncertainty of measurement for the declared results,
- Indication of coverage factor k, with the comment about the probability of coverage,

- For customers with less knowledge (about the topic), as applied, the following advices could be useful concerning:
    - the definition of indication error,
    - how to correct the readings in use by subtracting the corresponding errors.

8.4 Additional information

Additional information can be added to the certificate without being part of itself, additional information about the expected uncertainty of measurement in use, including the conditions under which it is applicable.

Where the errors are considered to obtain the correction, the following equation can be used:

$$\rho_u = R - E(R) \pm U(\rho_u) \qquad (8.1)$$

where,

ρ_u	Liquid density measured with the calibrated hydrometer in use, kg m^{-3}
R	Indication of the instrument in use. R is used in place of I to differentiate it from the indications during the calibration, kg m^{-3}
$E(R)$	Error of indication in use, see (9.2).
$U(\rho_u)$	Expanded uncertainty of the measured density with the calibrated instrument, kg m^{-3}

or where errors are included in the "overall uncertainty of the density $U_{gl}(\rho_u)$, and could be used the following relationship:

$$\rho_u = R \pm U_{gl}(\rho_u) \qquad (8.2)$$

It would have to enunciate the declaration on the expanded uncertainty associated with the resulting values of the equation with a confidence level of at least 95 %.

Optional:
Where applicable, a conformity declaration can be made for any existing specification (p.je. ISO 649), and a range of validity. In this way, the statement could be of the form:

$$\rho_u = R \pm Tol \qquad (8.3)$$

with,

Tol Tolerance, maximum permissible error according to the serial of the instrument, as long as the conformity of the instrument has been evaluated with respect to the corresponding requirements, see formula (4.1)

And this could be given in addition to the results of measurement, or as separate statement, with reference to the measurement results declared to be retained in the calibration laboratory.

The declaration may be accompanied by a comment indicating that all measurement results in addition to the corresponding expanded uncertainties are within the specification limits.

9 MEASUREMENT OF LIQUIDS DENSITY BY USING A CALIBRATED HYDROMETER

In the industry, it is necessary to measure the density of liquids used in processes of transformation, so that hydrometers are used for determination of liquids density, by what we have to bear in mind the following:

- The indications obtained in the normal use of the instrument for the measurement of liquids are not the same as those that were obtained during calibration,
- The measurement process can be different from the calibration procedure, e.g. surely only a reading for each indication, not multiple readings to obtain the average value,
- The measuring/operation conditions may be different (temperature, temperature of the liquid, etc.) from the calibration,

9.1 Measurement of liquid density with a calibrated hydrometer

The value of density of a liquid being measured with a calibrated hydrometer would be obtained by the following expression

$$\rho_u = R - E(R) - \varepsilon_{du} \qquad (9.1)$$

ε_{du} Error due to the resolution on the use of the hydrometer, this resolution may be different to that determined by the calibration laboratory, considering that during calibration tools could be used that does not necessarily have the user or not with the same characteristics, e.g. vision camera, kg m^{-3}

The indication error in use of the instrument is the sum of the errors due to the height of the meniscus, the difference of temperature, the difference in surface tension of the liquid under test in relation to the conditions which are assumed for the calibration of the instrument and the possible drift from the error of indication of the hydrometer since its calibration,

$$E(R) = E(I) + \varepsilon_h + \varepsilon_{\Delta t} + \varepsilon_{\Delta\gamma} + \varepsilon_{drift\ h} \tag{9.2}$$

Where:

ε_h	Error of height of the meniscus, this occurs because in opaque liquids the hydrometer indication must be read at the top of the meniscus and not at the level of the mirror of the fluid, kg m^{-3}
$\varepsilon_{\Delta t}$	Error due to the temperature difference that may exist between the reference temperature of calibration of the hydrometer and the measuring temperature in use, kg m^{-3}
$\varepsilon_{\Delta\gamma}$	Error due to the difference in surface tension that may exist between the surface tension of reference (for use) at which the hydrometer was calibrated and the surface tension of the liquid in which the hydrometer is being used (use of the instrument), kg m^{-3}
$\varepsilon_{drift\ h}$	Error due to the possible drift from the error of the hydrometer since its calibration, kg m^{-3}

9.1.1 Error due to resolution in use of the instrument

The error due to the indication of density with the calibrated hydrometer is estimated with zero mean and uncertainty assuming a uniform probability distribution for an interval during which it can be assumed the indication of the hydrometer will be found,

$$\varepsilon_{du} = 0 \pm \frac{d_u}{\sqrt{12}} \tag{9.3}$$

d_u Resolution of the instrument in use, kg m^{-3}

9.1.2 Error due to the height of the meniscus

The error due to the meniscus height, when the indication cannot be read at the level of the liquid (slightly below the mirror of the liquid), is calculated as:

$$\varepsilon_h = \left(1 - \sqrt{1 + \frac{2 \cdot g \cdot D^2 \cdot R}{\gamma}}\right) \cdot \frac{\Delta\rho \cdot \gamma}{g \cdot \Delta l \cdot D \cdot R} \tag{9.4}$$

where,

R	Indication of density to the upper level of the meniscus, kg m^{-3}
γ	Surface tension of the fluid under measurement, N m^{-1}
$\Delta\rho$	Scale range of the hydrometer (scale division), kg m^{-3}
Δl	Spacing between marks on the scale of the hydrometer, m
D	Stem diameter, m
g	Acceleration due to local gravity in the place of measurement, m s^{-2}

The variance associated with the error of height of the meniscus is calculated as:

$$\begin{aligned} u^2(\varepsilon_h) = & \left[\frac{D \cdot \Delta\rho}{\gamma \cdot \Delta l \cdot \sqrt{1 + \frac{2 \cdot D^2 \cdot g \cdot R}{\gamma}}} + \frac{\Delta\rho - \Delta\rho \cdot \sqrt{1 + \frac{2 \cdot D^2 \cdot g \cdot R}{\gamma}}}{D \cdot g \cdot \Delta l \cdot R}\right]^2 \cdot u^2(\gamma) \\ & + \left[\frac{\gamma \cdot \Delta\rho \cdot (-1 + \sqrt{1 + \frac{2 \cdot D^2 \cdot g \cdot R}{\gamma}})}{D^2 \cdot g \cdot \Delta l \cdot R \cdot \sqrt{1 + \frac{2 \cdot D^2 \cdot g \cdot R}{\gamma}}}\right]^2 \cdot u^2(D) \\ & + \left[\frac{\gamma \cdot \Delta\rho \cdot (-1 + \sqrt{1 + \frac{2 \cdot D^2 \cdot g \cdot R}{\gamma}})}{D \cdot g \cdot \Delta l \cdot R \cdot}\right]^2 \cdot u^2(\Delta l) \\ & + \left[\frac{\Delta\rho \cdot (\gamma \cdot D^2 \cdot g \cdot R \cdot -\gamma \cdot \sqrt{1 + \frac{2 \cdot D^2 \cdot g \cdot R}{\gamma}})}{D \cdot g^2 \cdot \Delta l \cdot R \cdot \sqrt{1 + \frac{2 \cdot D^2 \cdot g \cdot R}{\gamma}}}\right]^2 \cdot u^2(g) \end{aligned} \tag{9.5}$$

An alternative is to estimate a maximum and minimum value of error due to the height of the meniscus, and based on these limit values to estimate the standard uncertainty associated with this error assuming a uniform probability distribution,

$$u(\varepsilon_h) = \frac{\varepsilon_{h\ max} - \varepsilon_{h\ min}}{\sqrt{12}} \tag{9.6}$$

9.1.3 Error due to the temperature difference

The error due to the difference between the measuring temperature with respect to the reference temperature (in which was calibrated the hydrometer), is calculated with the following expression:

$$\varepsilon_{\Delta t} = \alpha \cdot R \cdot (t - t_0) \tag{9.7}$$

where,

α	Volumetric expansion coefficient of the glass (manufacturing material of the hydrometer), usually 25 x 10^{-6} °C^{-1}
R	Indication of the hydrometer in use, kg m^{-3}
t	Temperature of measurement, °C
t_0	Reference temperature in which was calibrated the instrument, °C

The variance associated with the temperature error is calculated as

$$u^2(\varepsilon_{\Delta t}) = R^2 \cdot (t - t_0)^2 \cdot u^2(\alpha) + \alpha^2 \cdot R^2 \cdot u^2(t) \tag{9.8}$$

There is a possibility of estimating the standard uncertainty associated with this error by estimating maximum and minimum limits of this value and assuming a uniform probability distribution of this range,

$$u^2(\varepsilon_{\Delta t}) = \frac{\varepsilon_{\Delta t\,max} - \varepsilon_{\Delta t\,min}}{\sqrt{12}} \tag{9.9}$$

9.1.4 Error due to difference in surface tension of the liquid under measurement and surface tension for which the instrument was calibrated

The error due to the difference in surface tension that exists between the surface tension of the fluid under measurement and the tension of reference (for which the hydrometer was calibrated), is calculated with the following formula,

$$\varepsilon_{\Delta \gamma} = R \cdot \pi \cdot D \cdot (\gamma_x - \gamma_u) \tag{9.10}$$

with

R	Indication of the hydrometer in use, kg m^{-3}
D	Diameter of the stem, m
γ_x	Surface tension of reference (value at which the instrument was calibrated), Nm
γ_u	Surface tension of the liquid under test, Nm

The variance associated with the error of measuring a liquid whose surface tension differs from the surface tension of reference is calculated as,

$$u^2(\varepsilon_{\Delta\gamma}) = [R \cdot \pi \cdot (\gamma_x - \gamma_u)]^2 \cdot u^2(D) + (R \cdot \pi \cdot D)^2 \cdot u^2(\gamma_u) \tag{9.11}$$

Or through the estimation of upper and lower limits of the value of this error, calculate the standard uncertainty,

$$u^2(\varepsilon_{\Delta\gamma}) = \frac{\varepsilon_{\Delta\gamma\, max} - \varepsilon_{\Delta\gamma\, min}}{\sqrt{12}} \tag{9.12}$$

9.1.5 Error due to the possible drift of the indication error of the hydrometer

In general the hydrometers are very stable instruments that with good maintenance (care and cleaning) their certified value can be preserved quite well, the problem is their construction since they are very fragile and only a few may have a history of their indication error, and based on this to estimate the drift value that could have the hydrometer after its calibration.

When applicable, the error associated with the drift of the instrument after its calibration can be estimated by the analysis of their previous calibrations (a possible tool are the control charts). The user should estimate a maximum drift value that the hydrometer could present at the moment of using the equipment and which considers the time elapsed since its last calibration. The error due to the drift is estimated with zero mean but is associated with a value of uncertainty as function of the maximum estimated drift value, as

$$\varepsilon_{drift\ h} = 0 \pm \frac{Drift_{\ h}}{\sqrt{12}} \tag{9.13}$$

$Drift_{\ h}$ Estimated maximum drift for the hydrometer since the calibration date, kg m^{-3}

9.2 Uncertainty of density measurement with the calibrated hydrometer

The uncertainty of the measurement of density with the calibrated hydrometer is obtained from the measurement model, replacing the equation (9.2) in (9.1).

$$\rho_u = R - E(I) - \varepsilon_h - \varepsilon_{\Delta t} - \varepsilon_{\Delta\gamma} - \varepsilon_{du} - \varepsilon_{drift\ h} \tag{9.14}$$

The variance of the measurement outcome of density of the liquid (use of the hydrometer), is calculated with the following formula,

$$u^2(\rho_u) = u^2(R) + u^2\big(E(I)\big) + u^2(\varepsilon_h) + u^2(\varepsilon_{\Delta t}) + u^2(\varepsilon_{\Delta\gamma}) + u^2(\varepsilon_{du}) + u^2\big(\varepsilon_{drift\ h}\big) \tag{9.15}$$

The expanded uncertainty associated with the measurement of density of the liquid with the calibrated hydrometer is obtained by multiplying the combined standard uncertainty (9.15) by a coverage factor k to increase the confidence level, usually $k = 2$, which equals approximately 95 % confidence level.

$$U(\rho_u) = k \cdot u(\rho_u) \tag{9.16}$$

9.3 Global uncertainty associated to the indication in use of a calibrated hydrometer

Depending on the required uncertainty in the measurement of liquids density with the calibrated hydrometer, the user could measure the density of the liquid directly from the indication of the calibrated hydrometer and without applying any correction to get the value of density of the liquid under test. The uncertainty associated with this measurement should be a “global” uncertainty that includes within the uncertainty value all the necessary information to compensate for the fact of not applying the pertinent corrections.

The global expanded uncertainty (approx. 95 % of confidence level), can be estimated using the following expression:

$$U_{gl}(\rho_u) = 2 \cdot \left[\left(E(R)\right)^2 + \left(\frac{d_u}{\sqrt{12}}\right)^2\right]^{1/2} \tag{9.17}$$

The uncertainty calculated by the formula (9.17) results in a higher value to that which is obtained by applying the pertinent corrections, whose associated uncertainty is obtained by the formula (9.16), however for some applications, it is sufficient.

For an instrument that meets the requirements of the ISO 649, $E(R) \leq \frac{2}{3} mpe$, therefore, considering this maximum error value as a interval cua probability distribution is uniform,

$$U_{gl}(\rho_u) = 2 \cdot \left[\left(\frac{2 \cdot mpe}{3 \cdot \sqrt{3}}\right)^2 + \left(\frac{d_u}{\sqrt{12}}\right)^2\right]^{1/2} \tag{9.18}$$

10 REFERENCES

[1] JCGM 200:2008 International vocabulary of metrology – Basic and general concepts and associated terms (VIM).

[2] ISO 649-1:1981 Laboratory glassware – Density hydrometers for general purposes—Part 1: Specification.

[3] Becerra Luis O., Peña Luis M., Daued Arturo A., Incertidumbre requerida en la evaluación de la conformidad de instrumentos para la medición de densidad: hidrómetros y densímetros de tipo oscilatorio, Simposio de Metrología 2012

[4] ISO 387:1977 Hydrometers – Principles of construction and adjustment

[5] OIML R111-1:2004 Weights of classes E_1, E_2, F_1, F_2, M_1, $M_{1–2}$, M_2, $M_{2–3}$ and M_3, Part 1: Metrological and technical requirements

[6] Lorefice Salvatore and Malengo Andrea, Calibration of hydrometers; Measurement Science and technology 17 (2006) 2560-2566

[7] OIML G 14 Edition 2011 (E), Guide Density measurement

[8] NMX-EC-17025-IMNC-2006, Evaluación de la conformidad Requisitos generales para la competencia de los laboratorios de ensayo y de calibración.

[9] CUCKOW- Calibration of Reference Standard Hydrometers (Journal of the Society of Chemical Industry, 68, February 1949).

[10] JCGM 100:2008 Evaluation of measurement data – Guide to the expression of uncertainty in measurements, GUM 1995 with minor corrections.

[11] ISO/IEC 17025:2005 General requirements for the competence of testing and calibration laboratories.

[12] Gupta S V, “Practical Density Measurement and Hydrometry”, Institute of Physics Publishing, Series in Measurement Science and Technology, 2002.

[13] Tanaka M., Girard G., Davis R., Peuto A., Bignell N., Recommended table for the density of water between 0 °C and 40 °C based on recent experimental reports. Metrologia, 2001, 38, 301 – 309.

[14] Peña Luis M, Pedraza J. Carlos, Becerra Luis O., Galvan Carlos A, - A new image processing system for hydrometers calibration developed at CENAM – IMEKO 20th TC3, 3rd TC16 and 1st TC22 International Conference “Cultivating Metrology Knowledge”, 27th to 30th November, 2007, Merida Mexico.

[15] S Lorefice and A Malengo - An image processing approach to calibration of hydrometers – Metrologia 41 (2004) L7 – L10.

[16] Notas del Taller en Calibración de Hidrómetros del SIM realizado en Querétaro, Qro. México del 21 al 23 de Noviembre del 2006.
www.cenam.mx/myd

[17] A Picard, R S Davis, M Gläser and K Fujii – Revised formula for the density of moist air (CIPM 2007), Metrologia 45 (2008) 149- 155.

[18] Guía SIM de Calibración de densímetros de tipo oscilatorio en preparación.

[19] SIM Guidelines on the calibration of non-automatic weighing instruments, 2009, SIM MWG7/cg-01/v.00

[20] Le Système international d´unités, SI, 8e édition 2006. Bureau international des poids et mesures, Organisation intergouvemementale de la Convention du Mètre

[21] ISO 10012:2003 Measurement management systems -- Requirements for measurement processes and measuring equipment

[22] The International Association for the Properties of Water and Steam –Release on the IAPWS Formulation 1995 for the Thermodynamic Properties of Ordinary Water Substance for General and Scientific Use - Fredericia, Denmark, September 1996.
[23] ISO 3696:1987 Water for analytical use – Specification and test methods.

ANNEX A – TRACEABILITY

In hydrometers calibration, where there are more than one input quantity in the measurement model (see chapter 6), each input quantity should itself be metrologicallly traceable.

The effort involved in establishing metrological traceability for each input quantity value should be commensurate with its relative contribution to the measurement result.

Below is a generalized example to develop a scheme of traceability in calibration of the hydrometers by Cuckow method.

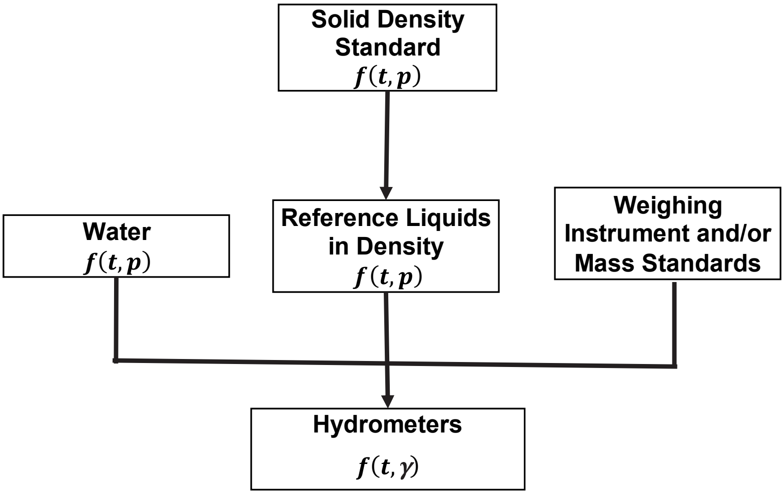

Figure A.1 Schematic of traceability in the calibration of hydrometers by the Cuckow method. Calibration of hydrometers are associated to a temperature and to a surface tension, where they normally will be used.

ANNEX B. SUGGESTIONS FOR THE ESTIMATION OF THE DENSITY OF AIR AND WATER

The air and water are two fluids commonly used to adjust the oscillation-type density meters. The density of air can be determined using the formula CIPM-2007 [17], from measurements of temperature, pressure and humidity of the location, which offers the best accuracy. If values of less accuracy are required, this is, with greater uncertainty, there are two approaches to the CIPM formula-2007 [19].

For the density of water, the CIPM recommends the use of the formula developed by M. Tanaka et al. [13]. The value of the density of water can be used for quality assurance in the measurements, and this value depends on the purity of the water, which is related to its resistivity/conductivity.

Note: *In Annex B, the symbol T is used to indicate the temperature in kelvins, and the symbol t, is used to indicate the temperature in Celsius degrees.*

B.1 Equation for the calculation of the density of air

The equation of greater accuracy to determine the density of air with the best uncertainty (as far as the contribution of the formula refers), is recommended by the CIPM[5]. The uncertainty of the density of air, will depend on the characteristics of the instruments for measuring environmental conditions, the calibration of these, and the stability of the environmental conditions.

However for some calibrations, depending on the uncertainty required, it is possible to use a simplified version, which has associated uncertainty (to the formula) greater than the formula of the CIPM 2007 [19].

B.1.1 Simplified version of the equation CIPM-2007, exponential version

$$\rho_a = \frac{0.348\ 48p - 0.009 h_r\, exp(0.061t)}{273.15 + t} \tag{B.1}$$

with

ρ_a air density in kg m^{-3}

p barometric pressure in hPa

h_r relative humidity of air in %

t air temperatura in °C

---

[5] The intervals of pressure, temperature and humidity recommended for the implementation of the equation CIPM-2007 are:

600 hPa $\leq p \leq 1100$ hPa

15 °C $\leq t \leq 27$ °C

$0 \leq h_r \leq 1$

The equation gives results with $u_{\rho_a,form}/\rho_a \leq 2.4\times10^{-4}$ under the following environmental conditions (uncertainties of measurement of p, h_r and t not included):

$$600 \text{ hPa} \leq p \leq 1100 \text{ hPa} \quad \text{(B.2)}$$
$$20\ \% \leq h_r \leq 80\ \%$$
$$15\ °\text{C} \leq t \leq 27\ °\text{C}$$

B.1.2 Simplified version of the equation CIPM-2007, normal version

$$\rho_a = \frac{0.348\ 444p - h_r(0.002\ 52t - 0.020\ 582)}{273.15 + t} \quad \text{(B.3)}$$

With the same symbols for B.1.

The equation gives results with $\Delta\rho_{a,form} \leq$ 0.001 41 kg/m³ under the following environmental conditions (not including the uncertainties of measurement of p, h_r y t):

$$600 \text{ hPa} \leq p \leq 1100 \text{ hPa}$$
$$20\ \% \leq h_r \leq 80\ \%$$
$$15\ °\text{C} \leq t \leq 27\ °\text{C}$$

$\Delta\rho_{a,form}$ is the difference between values obtained from this equation and the corresponding values of the equation CIPM. Therefore, the relative variance of the equation is given by:

$$\hat{w}^2(\rho_{a,form}) = (2.2x10^{-5})^2 + \frac{\left(\frac{0.00141\ kg\,m^{-3}}{1.2\ kg\,m^{-3}}\right)^2}{3} = 4.61\times10^{-7} \quad \text{(B.4)}$$

And the relative standard uncertainty of the equation is,

$$\hat{w}(\rho_{a,form}) = 6.79\times10^{-4} \quad \text{(B.5)}$$

B.2 Equation for the density of water

The equation of M. Tanaka et al [13] offers values of the density of the water, ρ_w free of dissolved air, depending on its temperature t, and valid in the range 0 °C to 40 °C to a reference pressure of 101 325 Pa. The uncertainty of the formula of M. Tanaka has a relative uncertainty of 4.5×10^{-7} , which is combined with the other contributions (e.g. temperature and pressure, among others).

As not all water samples has the same isotopic content, and in order to reach the highest accuracy in the calculation of the water density, the Tanaka equation has the possibility to calculate the water density for the specific isotopic content of the water available.

Since the water is slightly compressible and can contain dissolved air, the equation provides corrections because of these two effects.

When it is required to know the density of pure water, outside the ranges of temperature and pressure which covers the equation of M. Tanaka [16], it can be used the formula known as IAPWS-95 [22].

For some applications, depending on the required uncertainty, it can be used another approach, whose calculation is simpler but with a greater uncertainty.

B.2.1 Alternative equation for the water density, polynomial of fourth degree

The following simplified expression can be used in the range of 1 °C to 40 °C, and at the atmospheric pressure of 101,325 kPa,

$$\rho_w(kg\ m^{-3}) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 \tag{B.6}$$

where,

ρ_w is the density of the water at the working temperature (kg m^{-3})

$a_0 = 999.84\ \mathrm{kg\ m^{-3}}$

$a_1 = 6.6054 \times 10^{-2}\ \mathrm{°C^{-1} kg\ m^{-3}}$

$a_2 = -8.7291 \times 10^{-3}\ \mathrm{°C^{-2} kg\ m^{-3}}$

$a_3 = 7.5787 \times 10^{-5}\ \mathrm{°C^{-3} kg\ m^{-3}}$

$a_4 = -4.5058 \times 10^{-7}\ \mathrm{°C^{-4} kg\ m^{-3}}$

Formula B. 2.1 -1 provides results with $\Delta\rho_{w,form} \leq$ 0.005 1 kg/m³ in the range of 1 °C to 40 °C. $\Delta\rho_{w,form}$ is the difference between values obtained from this equation and the corresponding values in the equation of M. Tanaka, therefore, the relative variance of the equation is given by

$$\hat{w}^2(\rho_{w,form}) = (4.51 \times 10^{-7})^2 + \frac{\left(\frac{0.005\ 1\ kg\,m^{-3}}{998.2\ kg\,m^{-3}}\right)^2}{3} = 8.905 \times 10^{-12} \qquad \text{(B.7)}$$

and the relative standard uncertainty is,

$$\hat{w}(\rho_{w,form}) = 3 \times 10^{-6} \qquad \text{(B.8)}$$

The uncertainty of the density of water is obtained from the combination of this contribution (due to the formula used), with the contributions due to the temperature and pressure of the water and, if this is the case, the contribution due to the correction by dissolved gases in the water.

For the range of temperature between 15 °C to 25 °C, the density differences are $\Delta\rho_{w,form} \leq$ 0.000 5 kg m^{-3}, therefore the relative uncertainty associated to this equation for this range of temperature is,

$$\hat{w}(\rho_{w,form}) = 5.4 \times 10^{-7} \qquad \text{(B.9)}$$

The corrections due to the pressure and temperature, as well as the correction due to dissolved gases in the water are described in the article by M. Tanaka [13].

ANNEX C. COVERAGE FACTOR k FOR THE EXPANDED UNCERTAINTY OF MEASUREMENT

***Note:** in this Appendix the general symbol y is used for the result of the measurement, not as a particular quantity, as an indication, an error, a value of the mass of a heavy object, etc.*

C.1 Objective

The coverage factor k must be chosen for all the cases that the expanded uncertainty of measurement have a coverage probability of approximately 95 %.

C.2 Basic conditions for the implementation of

A factor of k = 2 applies when the following conditions are met:

A) A normal distribution can be assigned to the resulting estimate y also $u(y)$ is sufficiently reliable.

B) It can be assumed a normal distribution when several components of uncertainty (e.g. N ≥ 3), each one derived from distributions of "common behaviours" (normal, rectangular or similar), contribute to $u(y)$ in comparable quantities.

> ***Note:** this implies that none of the contributions with different distribution to the normal is a dominant value as defined in C.3.2.*

The **sufficient reliability** depends on the effective degrees of freedom. This criterion is met if no contribution Type A of $u(y)$ is based on fewer than 10 observations.

C.3 Determining *k* for other cases

In any of the following cases the expanded uncertainty is:

$$U(y) = ku(y) \tag{C.1}$$

C.3.1 Distribution assumed as normal

Where the distribution of the estimated for the output variable y can be assumed as a normal distribution, but $u(y)$ is not reliable enough - see chapter C. 2 - then the effective degrees of freedom ν_{eff} have to be determined using the formula of Welch-Satterthwaite, and the value of k > 2 is obtained from the corresponding table.

$$\nu_{eff} = \frac{u^4(y)}{\sum_{i=1}^{N} \frac{u^4(x_i)}{\nu_i}} \tag{C.2}$$

Where

ν_{eff} are de effective degree of freedom associated to $u(y)$
$u(y)$ is the combined standard uncertainty associated to measurand Y
$u(x_i)$ is the standard uncertainty associated to the input quantity i
ν_i are the degree of freedom associated to the uncertainty of the input quantity i

From the corresponding table, take $k_p = t_p(\nu_{eff})$ and calculate $U_p(y) = k_p u(y)$, to obtain the desired level of confidence.

Note: *The selection of the $t_p(\nu)$ is based on the degrees of freedom and fraction p from the t-distribution table.*

C.3.2 Non-normal distribution

It may be obvious in a given situation that $u(y)$ contains a component of type B uncertainty of $u_1(y)$ that has a contribution with non-normal distribution, e.g. rectangular or triangular, which is considerably higher than the rest of the components. In this case, $u(y)$ is divided in the part (possibly dominant) u_1 and in u_R = square root of $\sum u_j^2$ with j ≥ 2, the combined standard uncertainty includes the remaining contributions, see [1]. If u_R ≤ $0.3u_1$, then u_1 is considered as "dominant" and the distribution of y is considered basically identical to that of the dominant contribution.

The coverage factor is chosen according to the shape of the distribution of the dominant component:

For the trapezoidal distribution with < 0.95 :

(β = side parameter, rate of lower side to larger side of the trapezoid)

$$k = \left\{1 - \sqrt{[0.05(1-\beta^2)]}\right\} / \sqrt{[(1+\beta^2)/6]} \tag{C.3}$$

- for a rectangular distribution (β = 1): k = 1.65
- for a triangular distribution (β = 0): k = 1.90
- for a type distribution U: k = 1.41

The dominant component may in turn be composed of two dominant components $u_1(y)$, $u_2(y)$, e.g. two rectangles forming a trapezoid, in which case u_R will be determined from the remaining u_j with $j \geq 3$.

ANNEX D – EXAMPLES

The examples presented in this Annex shows some of the different ways of how to apply properly the rules contained in this guide. It is not intended to indicate preference of a procedure against another whose example is not submitted.

D1 Example: Calibration of hydrometer series L20, interval 1 480 kg m^{-3} to 1 500 kg m^{-3}.

Below is the calibration data of a hydrometer in the range of 1 480 to 1 500 kg m^{-3}, L20 series.

D1.1 Data from hydrometer and air weighing

Hydrometer		
Interval	1 480 kg m^{-3} - 1 500 kg m^{-3} @ 20 °C.	
Scale division	0.20	kg m^{-3}
Resolution	0.04	kg m^{-3}
Series	L20	
Maximum permissible error, mpe	0.2	kg m^{-3}
Required Uncertainty, k = 2	0.067	kg m^{-3}
Stem Diameter, D	0.004 3	m
$u\ (D)$, k=1	0.000 2	m
Thermal exp. coeff. α_{glass}	9.9E-06	°C^{-1}
$u(\alpha_{glass})$, k=1	1.00E-07	°C^{-1}
Local Gravity, g	9.780 8	m s^{-2}
$u\ (g)$, k = 1	0.000 5	m s^{-2}

Air weighing		
Air density, ρ_a	0.961 78	kg m^{-3}
$u(\rho_a)$, k=1	0.000 77	kg m^{-3}
Air temperature (t_a)	20.5	°C
$u(t_a)$, k=1	0.2	°C
Mass difference, Δm	1.10E-06	kg
Standard dev., $s(\Delta m)$	2.00E-07	kg
Number of measurements, n	3	
Resolution of the weighing inst., d	1.00E-07	kg
Mass of the standard in air, m_s	0.287 361 1	kg
$u(m_s)$, k=1	6.0E-07	kg

D1.2 Liquid of reference

Weighing in the liquid		
Density, ρ_{liq}	768.490	kg m^{-3}
$u(\rho_{liq})$, k=1	0.007	kg m^{-3}
Surface tension, γ_{liq}	0.027 0	N m^{-1}
$u(\gamma_{liq})$, k =1	0.001 5	N m^{-1}
Temperature, t_{liq}	20.00	°C
$u(t_{liq})$, k=1	0.06	°C

D1.3 Weighing in the liquid of reference, for the three nominal values

	Nominal Value 1		**Nominal Value 2**		**Nominal Value 3**	
Nominal Value	1 498	kg m^{-3}	1 490	kg m^{-3}	1 482	kg m^{-3}
Surface Tension. (in use), γ_x	0.075	N m^{-1}	0.075	N m^{-1}	0.075	N m^{-1}
Mass difference, Δm	-1.165E-04	kg	-1.900E-05	kg	8.19E-05	kg
Standard dev., $s(\Delta m)$	5.00E-07	kg	5.00E-07	kg	5.00E-07	kg
Number of measurements, n	3		3		3	
Balance resolution, d	1.00E-07	kg	1.00E-07	kg	1.00E-07	kg
Mass standard. Liquid $m_{s(liq)}$	0.140 135	kg	0.139 245	kg	0.138 344	kg
Stand. Uncert. $u(m_{s(liq)})$	1.9E-07	kg	2.6E-07	kg	2.5E-07	kg
Air Density, ρ_a	0.948 40	kg m^{-3}	0.947 57	kg m^{-3}	0.946 62	kg m^{-3}
Stand. Uncert. $u(\rho_a)$, k=1	0.000 77	kg m^{-3}	0.000 77	kg m^{-3}	0.000 77	kg m^{-3}

D1.4 Nominal Value 1: Uncertainty budget for the indication error in 1 498 kg m^{-3}

Source	Value	Variability	Distribution	Standard uncertainty	Sensitivity coefficient	Contribution
Hydrometer indication , $I(\rho)$	1 498.000 kg m^{-3}	0.003 kg m^{-3}	normal, k=1	0.003 kg m^{-3}	1	3.000E-03 kg m^{-3}
Density at the mark, ρ_x	1 498.019 kg m^{-3}	0.051 kg m^{-3}	normal, k=2	0.025 kg m^{-3}	1	2.50E-02 kg m^{-3}
Resolution error of the hydrometer, ε_d	0.000 kg m^{-3}	0.040 kg m^{-3}	rectangular	0.012 kg m^{-3}	1	1.20E-02 kg m^{-3}
					Combined standard uncertainty	0.028 kg m^{-3}
Indication error, $E(I)$	**-0.019 kg m^{-3}**				**Expanded uncertainty, k=2**	**0.056 kg m^{-3}**

D1.5 Uncertainty budget of density at the mark level

Source	**Value**	**Variability**	**Distribution**	**Standard uncertainty**	**Sensitivity coefficient**	**Contribution**
Density of the liquid of reference, ρ_L	768.490 kg m^{-3}	7.00E-03 kg m^{-3}	normal, k=1	7.00E-03 kg m^{-3}	1.950 5	1.365 3E-02 kg m^{-3}
Air density, ρ_a	0.96178 kg m^{-3}	7.70E-04 kg m^{-3}	normal, k=1	7.70E-04 kg m^{-3}	-0.950 486 25	-7.318 7E-04 kg m^{-3}
Apparent mass of the hydrometer in the air, m_a	0.287 396 75 kg	1.29E-07 kg	normal, k=1	1.29E-07 kg	-4925.100 5 m^{-3}	-6.357 7E-04 kg m^{-3}
Apparent Mass of the hydrometer in the fluid, m_L	0.140 035 1 kg	2.94E-07 kg	normal, k=1	2.94E-07 kg	10109.1908 m^{-3}	2.975 7E-03 kg m^{-3}
Temperature correction factor of the air, $f_{t,a}$	1.000 004 95	3.92E-12	normal, k=1	3.92E-12	-0.914 163 426 kg m^{-3}	-3.586 2E-12 kg m^{-3}
Temperature correction factor of the liquid, $f_{t,L}$	1	3.53E-13	normal, k=1	3.53E-13	1 498.932 982 kg m^{-3}	5.288 8E-10 kg m^{-3}
Surface tension of the liquid of reference, γ_L	0.027 0 N m^{-1}	0.001 5 N m^{-1}	normal, k=1	1.50E-03 N m^{-1}	14.027 747 36 kg m^{-2} N^{-1}	2.104 2E-02 kg m^{-3}
Surface tension of the liquid x, γ_x	0.075 N m^{-1}	cte.		---	---	---
Stem diameter, D	0.004 3 m	0.000 2 m	normal, k=1	2.00E-04 m	0.160 731 031 kg m^{-4}	3.214 6E-05 kg m^{-3}
Acceleration due to local gravity, g	9.780 8 m s^{-2}	0.000 5 m s^{-2}	normal, k=1	5.00E-04 m s^{-2}	-0.016 433 322 kg s^{2} m^{-4}	-8.216 7E-06 kg m^{-3}
					Combined standard uncertainty	0.025 kg m^{-3}
Density at mark level, ρ_x	**1 498.019 kg m^{-3}**				**Expanded uncertainty, k=2**	**0.051 kg m^{-3}**

D1.6 Nominal Value 2: Uncertainty budget for the indication error in 1 490 kg m^{-3}

Source	Value	Variability	Distribution	Standard uncertainty	Sensitivity coefficient	Contribution
Hydrometer indication , $I(\rho)$	1 490.000 kg m^{-3}	0.003 kg m^{-3}	normal, k=1	0.003 kg m^{-3}	1	3.00E-03 kg m^{-3}
Density at the mark, ρ_x	1 490.012 kg m^{-3}	0.050 kg m^{-3}	normal, k=2	0.025 kg m^{-3}	1	2.50E-02 kg m^{-3}
Resolution error of the hydrometer, ε_d	0.000 kg m^{-3}	0.040 kg m^{-3}	rectangular	0.012 kg m^{-3}	1	1.20E-02 kg m^{-3}
					Combined standard uncertainty	0.028 kg m^{-3}
Indication error, $E(I)$	**-0.012 kg m^{-3}**				**Expanded uncertainty, k=2**	**0.056 kg m^{-3}**

D1.7 Uncertainty budget of density at the mark level

Source	Value	Variability	Distribution	Standard uncertainty	Sensitivity coefficient	Contribution
Density of the liquid of reference, ρ_L	768.490 kg m^{-3}	7.00E-03 kg m^{-3}	normal, k=1	7.00E-03 kg m^{-3}	1.9401	1.358 0E-02 kg m^{-3}
Air density, ρ_a	0.961 78 kg m^{-3}	7.70E-04 kg m^{-3}	normal, k=1	7.70E-04 kg m^{-3}	-0.940 053 97	-7.238 4E-04 kg m^{-3}
Apparent mass of the hydrometer in the air, m_a	0.287 396 75 kg	1.29E-07 kg	normal, k=1	1.29E-07 kg	-4 845.093 1 m^{-3}	-6.254 4E-04 kg m^{-3}
Apparent Mass of the hydrometer in the liquid, m_L	0.139 242 49 kg	2.94E-07 kg	normal, k=1	2.94E-07 kg	10 001.578 32 m^{-3}	2.944 0E-03 kg m^{-3}
Temperature correction factor of the air, $f_{t,a}$	1.000 004 95	3.92E-12	normal, k=1	3.92E-12	-0.904 1298 68 kg m^{-3}	-3.546 8E-12 kg m^{-3}
Temperature correction factor of the liquid, $f_{t,L}$	1	3.53E-13	normal, k=1	3.53E-13	1 490.915 879 kg m^{-3}	5.260 5E-10 kg m^{-3}
Surface tension of the liquid of reference, γ_L	0.027 0 N m^{-1}	0.001 5 N m^{-1}	normal, k=1	1.50E-03 N m^{-1}	13.878 092 72 kg m^{-2} N^{-1}	2.081 7E-02 kg m^{-3}
Surface tension of the liquid x, γ_x	0.075 00 N m^{-1}	cte.	---	---	---	---
Stem diameter, D	0.004 3 m	0.000 2 m	normal, k=1	2.00E-04 m	0.161 885 811 kg m^{-4}	3.237 7E-05 kg m^{-3}
Acceleration due to local gravity, g	9.780 8 m s^{-2}	0.000 5 m s^{-2}	normal, k=1	5.00E-04 m s^{-2}	-0.016 551 387 kg s^{2} m^{-4}	-8.275 7E-06 kg m^{-3}
					Combined standard uncertainty	0.025 kg m^{-3}
Density at mark level, $\boldsymbol{\rho_x}$	**1 490.012 kg m^{-3}**				**Expanded uncertainty, k=2**	**0.050 kg m^{-3}**

D1.8 Nominal Value 3: Uncertainty budget for the indication error in 1 482 kg m^{-3}

Source	Value	Variability	Distribution	Standard uncertainty	Sensitivity coefficient	Contribution
Hydrometer indication , $I(\rho)$	1 482.000 kg m^{-3}	0.003 kg m^{-3}	normal, k=1	0.003 kg m^{-3}	1	3.00E-03 kg m^{-3}
Density at the mark, ρ_x	1 482.014 kg m^{-3}	0.050 kg m^{-3}	normal, k=2	0.025 kg m^{-3}	1	2.50E-02 kg m^{-3}
Resolution error of the hydrometer, ε_d	0.000 kg m^{-3}	0.040 kg m^{-3}	rectangular	0.012 kg m^{-3}	1	1.12E-02 kg m^{-3}
					Combined standard uncertainty	0.028 kg m^{-3}
Indication error, $E(I)$	**-0.014 kg m^{-3}**				**Expanded uncertainty, k=2**	**0.056 kg m^{-3}**

D1.9 Uncertainty budget of density at the mark level

Source	Value	Variability	Distribution	Standard uncertainty	Sensitivity coefficient	Contribution
Density of the liquid of reference, ρ_L	768.490 0 kg m^{-3}	7.00E-03 kg m^{-3}	normal, k=1	7.00E-03 kg m^{-3}	1.929 6	1.350 7E-02 kg m^{-3}
Air density, ρ_a	0.961 78 kg m^{-3}	7.70E-04 kg m^{-3}	normal, k=1	7.70E-04 kg m^{-3}	-0.929 634 22	-7.158 2E-04 kg m^{-3}
Apparent mass of the hydrometer in the air, m_a	0.287 396 75 kg	1.29E-07 kg	normal, k=1	1.29E-07 kg	-4 765.755 3 m^{-3}	-6.152 0E-04 kg m^{-3}
Apparent Mass of the hydrometer in the fluid, m_L	0.138 442 28 kg	2.94E-07 kg	normal, k=1	2.94E-07 kg	9 894.667 364 m^{-3}	2.912 6E-03 kg m^{-3}
Temperature correction factor of the air, $f_{t,a}$	1.000 004 95	3.92E-12	normal, k=1	3.92E-12	-0.894 108 361	-3.507 5E-12 kg m^{-3}
Temperature correction factor of the liquid, $f_{t,L}$	1	3.53E-13	normal, k=1	3.53E-13	1 482.908 406	5.232 2E-10 kg m^{-3}
Surface tension of the liquid of reference, γ_L	0.027 00 N m^{-1}	0.001 50 N m^{-1}	normal, k=1	1.50E-03 N m^{-1}	13.729 418 96 kg m^{-2} N^{-1}	2.059 4E-02 kg m^{-3}
Surface tension of the liquid x, γ_x	0.075 00 N m^{-1}	cte.	---	---	---	---
Stem diameter, D	0.004 3 m	0.000 2 m	normal, k=1	2.00E-04 m	0.163 017 577 kg m^{-4}	3.260 4E-05 kg m^{-3}
Acceleration due to local gravity, g	9.780 8 m s^{-2}	0.000 5 m s^{-2}	normal, k=1	5.00E-04 m s^{-2}	-0.016 667 101 kg s^{2} m^{-4}	-8.333 6E-06 kg m^{-3}
					Combined standard uncertainty	0.025 kg m^{-3}
Density at mark level, ρ_x	**1 482.014 kg m^{-3}**				**Expanded uncertainty, k=2**	**0.050 kg m^{-3}**

D1.10 Calibration results

Nominal Value	Indication Error	Unc. (k=2)	Temperature	Surface Tension
1 498 kg m^{-3}	-0.019 kg m^{-3}	0.056 kg m^{-3}	20 °C	0.075 N m^{-1}
1 490 kg m^{-3}	-0.012 kg m^{-3}	0.056 kg m^{-3}	20 °C	0.075 N m^{-1}
1 482 kg m^{-3}	-0.014 kg m^{-3}	0.056 kg m^{-3}	20 °C	0.075 N m^{-1}

Note: *For the calculation of the expanded uncertainty, it has been used a coverage factor k = 2, to obtain a level of confidence of 95 %, assuming a normal probability distribution based on the Central Limit Theorem. To have a better approximation of the whished confidence level, the coverage factor value could be obtained according to effective degrees of freedom (see Annex C).*

D2 Example: Calibration of hydrometer series M100, interval 800 kg m^{-3} to 900 kg m^{-3}.

Below is the calibration data of a hydrometer in the range of 800 to 900 kg m-3, M100 series

D2.1 Data from hydrometer and air weighing

Hydrometer		
Interval	800 kg m^{-3} - 900 kg m^{-3} @ 20 °C.	
Scale division	2	kg m-3
Resolution	0.2	kg m-3
Series	M100	
Maximum permissible error, mpe	2	kg m-3
Required Uncertainty, k=2	0.67	kg m-3
Stem diameter, D	0.006	m
u (D), k=1	0.000 2	m
Thermal expansion coeff. α_{glass}	9.9E-06	°C-1
$u(\alpha_{glass})$, k=1	1.00E-07	°C-1
Local Gravity, g	9.781	m s-2
u (g), k = 1	0.001	m s-2

Air weighing		
Air density, ρ_a	0.945	kg m-3
$u(\rho_a)$, k=1	0.003	kg m-3
Air temperature (t_a)	23.0	°C
$u(t_a)$, k=1	0.8	°C
Reading of the weighing inst. weighing the hydrometer, $R\ _{yd}$	0.143 4	kg
$s(R_{hyd})$	1.00E-06	kg
Number of measurements, n	4	
Resolution of the weighing inst., d	1.00E-06	kg
Indication error of the weighing inst., $E(I)$	5.00E-07	kg
$u(E(I))$, k=1	3.00E-06	kg

D2.2 Weighing in the reference liquid

Weighing in the liquid		
Density, ρ_{liq}	768.493	kg m-3
$u(\rho_{liq})$, k=1	0.007	kg m-3
Surface tension, γ_{liq}	0.027	N m-1
$u(\gamma_{liq})$, k =1	0.003	N m-1
Temperature, t_{liq}	20.00	°C
$u(t_{liq})$, k=1	0.05	°C

D2.3 Weighing in the liquid of reference, for the three nominal values

	Nominal Value 1		**Nominal Value 2**		**Nominal Value 3**	
Nominal Value	890	kg m^{-3}	850	kg m^{-3}	810	kg m^{-3}
Surface Tension. X (in use)), γ_x	0.029 5	N m^{-1}	0.027 5	N m^{-1}	0.025 5	N m^{-1}
Reading of the weighing inst. with the hydrometer Immersed, $R_{hyd(liq)}$	0.019 768	kg	0.013 938	kg	0.013 938	kg
$s(R_{hyd(liq)})$	2.00E-06	kg	2.00E-06	kg	2.00E-06	kg
Number of measurements, n	3		3		3	
Resolution of the weighing inst., d	1.00E-06	kg	1.00E-06	kg	1.00E-06	kg
Indication error of the weighing inst., $E(I)$	1.3E-07	kg	-1.0E-07	kg	-4.0E-08	kg
$u(E(I))$, k=1	4.0E-06	kg	4.0E-06	kg	4.0E-06	kg
Air density, ρ_a	0.940	kg m^{-3}	0.939	kg m^{-3}	0.938	kg m^{-3}
$u(\rho_a)$, k=1	0.003	kg m^{-3}	0.003	kg m^{-3}	0.003	kg m^{-3}

D2.4 Nominal Value 1: Uncertainty budget for the indication error in 890 kg m^{-3}

Source	Value	Variability	Distribution	Standard uncertainty	Sensitivity coefficient	Contribution
Hydrometer indication , $I(\rho)$	890.000 kg m^{-3}	0.050 kg m^{-3}	normal, k=1	0.050 kg m^{-3}	1	5.00E-02 kg m^{-3}
Density at the mark, ρ_x	891.198 kg m^{-3}	0.086 kg m^{-3}	normal, k=2	0.043 kg m^{-3}	1	4.30E-02 kg m^{-3}
Resolution error of the hydrometer, ε_d	0.000 kg m^{-3}	0.200 kg m^{-3}	rectangular	0.058 kg m^{-3}	1	5.80E-02 kg m^{-3}
					Combined standard uncertainty	0.09 kg m^{-3}
Indication error, $E(I)$	**-1.20 kg m^{-3}**				**Expanded uncertainty, k=2**	**0.18 kg m^{-3}**

D2.5 Uncertainty budget of density at the mark level

Source	Value	Variability	Distribution	Standard uncertainty	Sensitivity coefficient	Contribution
Density of the liquid of reference, ρ_L	768.493 kg m^{-3}	7.00E-03 kg m^{-3}	normal, k=1	7.00E-03 kg m^{-3}	1.159 9	8.119 1E-03 kg m^{-3}
Air density, ρ_a	0.9450 kg m^{-3}	3.00E-03 kg m^{-3}	normal, k=1	3.00E-03 kg m^{-3}	-0.159 836 326	-4.795 1E-04 kg m^{-3}
Apparent mass of the hydrometer in the air, m_a	0.143 417 44 kg	7.07E-07 kg	normal, k=1	7.07E-07 Kg	-984.435 5 m^{-3}	-6.960 6E-04 kg m^{-3}
Apparent Mass of the hydrometer in the liquid, m_L	0.019 770 45 kg	1.29E-06 kg	normal, k=1	1.29E-06 Kg	7 143.813 929 m^{-3}	9.221 8E-03 kg m^{-3}
Temperature correction factor of the air, $f_{t,a}$	1.000 029 7	6.28E-11	normal, k=1	6.28E-11	-0.151 073 394 kg m^{-3}	-9.489 9E-12 kg m^{-3}
Temperature correction factor of the liquid, $f_{t,L}$	1	2.45E-13	normal, k=1	2.45E-13	891.348 921 6 kg m^{-3}	2.184 0E-10 kg m^{-3}
Surface tension of the liquid of reference, γ_L	0.027 N m^{-1}	0.003 N m^{-1}	normal, k=1	3.00E-03 N m^{-1}	13.869 632 76 kg m^{-2} N^{-1}	4.160 9E-02 kg m^{-3}
Surface tension of the liquid x, γ_x	0.030 N m^{-1}	cte.		---	---	---
Stem diameter, D	0.006 m	0.000 2 m	normal, k=1	2.00E-04 m	-0.021 707 658 kg m^{-4}	-4.341 5E-06 kg m^{-3}
Acceleration due to local gravity, g	9.781 m s^{-2}	0.001 m s^{-2}	normal, k=1	1.00E-03 m s^{-2}	0.002 219 37 kg s^{2} m^{-4}	2.219 4E-06 kg m^{-3}
					Combined standard uncertainty	0.043 kg m^{-3}
Density at mark level, $\boldsymbol{\rho_x}$	**891.198 kg m^{-3}**				**Expanded uncertainty, k=2**	**0.086 kg m^{-3}**

D2.6 Nominal Value 2: Uncertainty budget for the indication error in 850 kg m^{-3}

Source	**Value**	**Variability**	**Distribution**	**Standard uncertainty**	**Sensitivity coefficient**	**Contribution**
Hydrometer indication , $I(\rho)$	850 kg m^{-3}	0.050 kg m^{-3}	normal, k=1	0.050 kg m^{-3}	1	5.00E-02 kg m^{-3}
Density at the mark, ρ_x	851.099 kg m^{-3}	0.080 kg m^{-3}	normal, k=2	0.040 kg m^{-3}	1	4.00E-02 kg m^{-3}
Resolution error of the hydrometer, ε_d	0.000 kg m^{-3}	0.200 kg m^{-3}	rectangular	0.058 kg m^{-3}	1	5.80E-02 kg m^{-3}
					Combined standard uncertainty	0.09 kg m^{-3}
Indication error, $E(I)$	**-1.10 kg m^{-3}**				**Expanded uncertainty, k=2**	**0.18 kg m^{-3}**

D2.7 Uncertainty budget of density at the mark level

Source	Value	Variability	Distribution	Standard uncertainty	Sensitivity coefficient	Contribution
Density of the liquid of reference, ρ_L	768.493 0 kg m^{-3}	7.00E-03 kg m^{-3}	normal, k=1	7.00E-03 kg m^{-3}	1.107 6	7.753 4E-03 kg m^{-3}
Air density, ρ_a	0.945 0 kg m^{-3}	3.00E-03 kg m^{-3}	normal, k=1	3.00E-03 kg m^{-3}	-0.107 594 066	-3.227 8E-04 kg m^{-3}
Apparent mass of the hydrometer in the air, m_a	0.143 417 44 kg	7.07E-07 kg	normal, k=1	7.07E-07 kg	-633.220 6 m^{-3}	-4.477 3E-04 kg m^{-3}
Apparent Mass of the hydrometer in the liquid, m_L	0.013 939 54 kg	1.29E-06 kg	normal, k=1	1.29E-06 kg	6 517.278 045 m^{-3}	8.413 0E-03 kg m^{-3}
Temperature correction factor of the air, $f_{t,a}$	1.000 029 7	6.28E-11	normal, k=1	6.28E-11	-0.101 704 459 kg m^{-3}	-6.388 7E-12 kg m^{-3}
Temperature correction factor of the liquid, $f_{t,L}$	1	2.45E-13	normal, k=1	2.45E-13	851.201 110 6 kg m^{-3}	2.085 7E-10 kg m^{-3}
Surface tension of the liquid of reference, γ_L	0.027 N m^{-1}	0.003 N m^{-1}	normal, k=1	3.00E-03 N m^{-1}	12.648 688 74 kg m^{-2} N^{-1}	3.794 6E-02 kg m^{-3}
Surface tension of the liquid x, γ_x	0.028 N m^{-1}	cte.	---	---	---	---
Stem diameter, D	0.006 m	0.000 2 m	normal, k=1	2.00E-04 m	-0.027 471 604 kg m^{-4}	-5.494 3E-06 kg m^{-3}
Acceleration due to local gravity, g	9.781 m s^{-2}	0.001 m s^{-2}	normal, k=1	1.00E-03 m s^{-2}	0.002 808 67 kg s^{2} m^{-4}	2.808 7E-06 kg m^{-3}
					Combined standard uncertainty	0.040 kg m^{-3}
Density at mark level, $\boldsymbol{\rho_x}$	**851.099 kg m^{-3}**				**Expanded uncertainty, k=2**	**0.080 kg m^{-3}**

D2.8 Nominal Value 3: Uncertainty budget for the indication error in 810 kg m^{-3}

Source	Value	Variability	Distribution	Standard uncertainty	Sensitivity coefficient	Contribution
Hydrometer indication , $I(\rho)$	810 kg m^{-3}	0.050 kg m^{-3}	normal, k=1	0.050 kg m^{-3}	1	5.00E-02 kg m^{-3}
Density at the mark, ρ_x	810.998 kg m^{-3}	0.072 kg m^{-3}	normal, k=2	0.036 kg m^{-3}	1	3.60E-02 kg m^{-3}
Resolution error of the hydrometer, ε_d	0 kg m^{-3}	0.200 kg m^{-3}	rectangular	0.058 kg m^{-3}	1	5.80E-02 kg m^{-3}
					Combined standard uncertainty	0.085 kg m^{-3}
Indication error, $E(I)$	**-1.00 kg m^{-3}**				**Expanded uncertainty, k=2**	**0.17 kg m^{-3}**

D2.9 Uncertainty budget of density at the mark level

Source	Value	Variability	Distribution	Standard uncertainty	Sensitivity coefficient	Contribution
Density of the liquid of reference, ρ_L	768.493 0 kg m^{-3}	7.00E-03 kg m^{-3}	normal, k=1	7.00E-03 kg m^{-3}	1.054 1	7.387 6E-03 kg m^{-3}
Air density, ρ_a	0.945 0 kg m^{-3}	3.00E-03 kg m^{-3}	normal, k=1	3.00E-03 kg m^{-3}	-0.055 347 879	-1.660 4E-04 kg m^{-3}
Apparent mass of the hydrometer in the air, m_a	0.143 417 44 kg	7.07E-07 kg	normal, k=1	7.07E-07 Kg	-310.705 4 m^{-3}	-2.196 9E-04 kg m^{-3}
Apparent Mass of the hydrometer in the liquid, m_L	0.007 530 84 kg	1.29E-06 kg	normal, k=1	1.29E-06 Kg	5 919.223 346 m^{-3}	7.641 0E-03 kg m^{-3}
Temperature correction factor of the air, $f_{t,a}$	1.000 029 7	6.28E-11	normal, k=1	6.28E-11	-0.052 331 812	-3.287 3E-12 kg m^{-3}
Temperature correction factor of the liquid, $f_{t,L}$	1	2.45E-13	normal, k=1	2.45E-13	811.050 281 5	1.987 3E-10 kg m^{-3}
Surface tension of the liquid of reference, γ_L	0.027 N m^{-1}	0.003 N m^{-1}	normal, k=1	3.00E-03 N m^{-1}	11.483 872 67 kg m^{-2} N^{-1}	3.445 2E-02 kg m^{-3}
Surface tension of the liquid x, γ_x	0.026 N m^{-1}	cte.	---	---	---	---
Stem diameter, D	0.006 M	0.000 2 m	normal, k=1	2.00E-04 m	-0.032 597 838 kg m^{-4}	-6.519 6E-06 kg m^{-3}
Acceleration due to local gravity, g	9.781 m s^{-2}	0.001 m s^{-2}	normal, k=1	1.00E-03 m s^{-2}	0.003 332 771 kg s^{2} m^{-4}	3.332 8E-06 kg m^{-3}
					Combined standard uncertainty	0.036 kg m^{-3}
Density at mark level, $\boldsymbol{\rho_x}$	**810.998 kg m^{-3}**				**Expanded uncertainty, k=2**	**0.072 kg m^{-3}**

D2.10 Calibration Results

Nominal Value	Indication Error	Unc. (k=2)	Temperature	Surface Tension
890 kg m^{-3}	-1.20 kg m^{-3}	0.18 kg m^{-3}	20 °C	0.0295 N m^{-1}
850 kg m^{-3}	-1.10 kg m^{-3}	0.18 kg m^{-3}	20 °C	0.0275 N m^{-1}
810 kg m^{-3}	-1.00 kg m^{-3}	0.17 kg m^{-3}	20 °C	0.0255 N m^{-1}

D.3 Example: Measurement of the density of an oil

Below is presented the measurement of density of an oil with same hydrometer from example D2 calibrated, the data of the hydrometer is found in the table D2.1.

D3.1 Data from the liquid under test and thermometer

Oil identification	ISO 68
Surface tension, , γ_{liq}	0.030 N m^{-1}
$u(\gamma_{liq})$, k =1	0.005 N m^{-1}
Density, ρ_{liq}	880 kg m^{-3}
Thermometer	
Resolution	0.1 °C
Indication error, $E(I)$	0.0 °C
$u(E(I))$, k=1	0.2 °C

D3.2 Hydrometer readings and temperature of the liquid under test

	Hydrometer indication	**Thermometer Indication**
R_1	879.5 kg m^{-3}	21.5 °C
R_2	879.5 kg m^{-3}	21.5 °C
R_3	879.6 kg m^{-3}	21.0 °C
R_4	879.5 kg m^{-3}	21.0 °C
Mean value, R	879.5 kg m^{-3}	21.3 °C
Standard Deviation, $s(\bar{R})$	0.05 kg m^{-3}	0.29 °C

D3.2 Uncertainty budget for the measurement of a fluid with a calibrated hydrometer

Source	Value	Variability	Distribution	Standard uncertainty	Sensitivity coefficient	Contribution
Hydrometer indication, R	879.525 kg m^{-3}	0.029 kg m^{-3}	normal, k=1	0.029 kg m^{-3}	-1	-2.90E-02 kg m^{-3}
Calibration error, $E(I)$	-1.20 kg m^{-3}	0.18 kg m^{-3}	normal, k=2	0.09 kg m^{-3}	-1	-9.00E-02 kg m^{-3}
Error due to the difference of temp, $\varepsilon_{\Delta t}$	0.010 9 kg m^{-3}	0.001 1 kg m^{-3}	normal, k=1	1.10E-03 kg m^{-3}	-1	-1.10E-02 kg m^{-3}
Error due to the meniscus height, ε_h	Negligible for the measurement of the density of clear liquids in which you can take the reading below the mirror of the liquid					
Error due to surface tension, $\varepsilon_{\Delta\gamma}$	0.017 kg m^{-3}	0.083 kg m^{-3}	normal, k=1	8.30E-02 kg m^{-3}	-1	-8.30E-02 kg m^{-3}
Error due to hydrometer resolution, ε_{du}	0.00 kg m^{-3}	0.67 kg m^{-3}	uniform	0.19 kg m^{-3}	-1	-1.93E-01 kg m^{-3}
Error due to hydrometer drift, $\varepsilon_{drift\ h}$	Negligible for a newly calibrated instrument					
					Combined standard uncertainty	0.23 kg m^{-3}
Liquid density, $\boldsymbol{\rho_u}$	**879.49 kg m^{-3}**				**Expanded uncertainty, k=2**	**0.46 kg m^{-3}**

D3.2 Expression of the outcome of the density of the liquid

The result of the measurement of the fluid is expressed as follows,

Liquid Density, ρ_u	**880.70 kg m^{-3}**	**Expanded Uncertainty, k=2**	**0.46 kg m^{-3}**	**Temperature**	**21.3 °C**

Considering a global uncertainty,

Liquid Density, ρ_u	**879.5 kg m^{-3}**	**Global Uncertainty, k=2**	**1.9 kg m^{-3}**	**Temperature**	**21.3 °C**

Tolerance Interval

Liquid Density, ρ_u Density at the level mark	**879.5 kg m^{-3}**	**Tolerance (±)**	**2.0 kg m^{-3}**	**Temperature**	**21.3 °C**

From the different ways to express the result of the liquid density under test, the declared values with an estimated global uncertainty or using the tolerance of the hydrometer associated with their series, the results of uncertainty values are much higher than if appropriate corrections are applied.